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An initial model for complex dynamics in electric power system blackouts

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Abstract

We define a model for the evolution of a long series of electric power transmission system blackouts. The model describes opposing forces which have been conjectured to cause self-organized criticality in power system blackouts. There is a slow time scale representing the opposing forces of load growth and growth in system capacity and a fast time scale representing cascading line overloads and outages. The time scales are coupled: load growth leads to outages and outages lead to increased system capacity. The opposing forces result in a dynamic equilibrium in which blackouts of all sizes occur. The model is a means to study the complex dynamics of this dynamic equilibrium. The Markov property of the model is briefly discussed. The model dynamic equilibrium is illustrated using initial results from the 73 bus IEEE reliability test system.

1. Introduction

Electric power transmission systems are large complex systems which operate near their capacity limits and exhibit cascading failures which often lead to blackouts. Any regularities or dynamics in series of blackouts of a nation's electrical power supply are likely to be of interest both because of the challenging complexity and size of the power system and its importance to the national infrastructure.

Recent analyses of time series of North American blackouts show evidence of long range correlations and probability distributions which suggests that the power system is exhibiting dynamics consistent with self-organized criticality [4, 5]. Moreover, [4] suggests a qualitative account of the processes which could give rise to self-organized criticality in a power system. The processes are a slow, secular load increase and the various engineering responses to blackouts. The slow, secular load increase tends to reduce the margins of transmission lines and hence increases the risk of cascading failure and blackouts. The various engineering responses to blackouts include improvements in operating policies, maintenance and equipment or controls and all these responses tend to increase the margins of transmission lines. It is conjectured in [4] that these opposing processes come to a dynamic equilibrium which is self-organized critical. The main purpose of this paper is to define a model detailing these processes to allow its complex system dynamics to be critically examined and better understood.

A self-organized critical system is one in which the nonlinear dynamics in the presence of perturbations organize the overall average system state near to, but not at, the state that is marginal to major disruptions [2, 8]. Self-organized critical systems are characterized by a spectrum of spatial and temporal scales of the disruptions that exist in remarkably similar forms in a wide variety of physical systems. One of the defining examples of self-organized criticality is the idealized running sandpile, which organizes itself to a critical gradient in which avalanches of all sizes occur. The blackout model is constructed with a broad analogy to the processes occurring in the sandpile in mind.

For this study of power system global complex dynamics, the model is "top-down" and represents the processes in greatly simplified forms, although the interactions between these processes still yield complex behavior. The simple representation of the processes is desirable both to study only the main interactions governing the complex dynamics and for pragmatic reasons of model tractability and simulation run time.

Intrinsic to this type of model are both slow and fast time scales. Load growth and responses to blackouts occur on a slow time scale of days to years and the cascading events causing blackouts occur in a fast time scale of minutes to hours. The slow dynamics are indexed by days so that load growth and responses to blackouts are updated daily, although the changes may be small.

The fast dynamics are cascading events. Events are the outage of a line or the limiting of flow on a transmission line to its maximum. Events can happen at any time but tend to be more more likely and more widespread at peak load when the network is most stressed. For simplicity, the daily peak load is chosen as representative of the loading during each day and the events are computed based on that peak load. Each day has the possibility of one cascade of events. The lines involved in the cascade are represented but the timing of events is neglected.

2. Slow dynamics

2.1. Network model

There are n buses and each bus is either a load or a generator. Let

$$P_{ik} = \text{real power injected at bus } i \text{ on day } k.$$
$$P_k = (P_{1k}, P_{2k}, P_{3k}, \ldots, P_{nk})^T$$

P_k is the vector of real power injections on day k. The power injections must satisfy the overall power balance $\sum_i P_{ik} = 0$.

There are m transmission lines. Let

$$F_{jk} = \text{real power flowing on line } j \text{ on day } k.$$
$$F_k = (F_{1k}, F_{2k}, F_{3k}, \ldots, F_{mk})^T$$

F_k is the vector of m line flows on day k. The line flows must satisfy

$$-F_{jk}^{max} \leq F_{jk} \leq F_{jk}^{max} \qquad j = 1, 2, \cdots, m \tag{1}$$

F_{jk}^{max} models a thermal or other type of limit on line j on day k.

Each transmission line is modeled as an inductance which may include the inductance of any equipment in the line such as transformers. Failure of this equipment is modeled as a failure of the line.

A DC (linearized) power flow model with no losses is assumed. It follows that there is a linear relation between power injections and line flows on day k:

$$F_k = AP_k \tag{2}$$

The matrix A represents the network constraints as explained in more detail in Appendix B.

On day 0 there are power injections P_0 and flows F_0 satisfying $F_0 = AP_0$ and $-F_{j0}^{max} \leq F_{j0} \leq F_{j0}^{max}$, $j = 1, 2, \cdots, m$.

2.2. Slow load increase

The slow load increase is modeled as

$$P_k = P_0 \prod_{\ell=1}^{k} \lambda_\ell \tag{3}$$

where $\lambda_1, \lambda_2, \lambda_3, \cdots$ are independent, identically distributed bounded continuous random variables in $[\lambda_{\min}, \lambda_{\max}]$ with mean value $\overline{\lambda}$ slightly larger than one. For example, $\overline{\lambda} = 1.00005$. The daily multiplication by λ_k represents a slowly increasing secular load trend with an additional random component.

Then the initial flows on day k are

$$F_k = AP_k = AP_0 \prod_{\ell=1}^{k} \lambda_\ell = F_0 \prod_{\ell=1}^{k} \lambda_\ell \tag{4}$$

These power injections and flows apply before any cascading events on day k.

2.3. Fractional overloads

Define the fraction of overload on line j on day k by

$$M_{jk} = \frac{F_{jk}}{F_{jk}^{max}} \tag{5}$$

A line with $M_{jk} < 1$ has margin remaining whereas a line with $M_{jk} > 1$ is overloaded. The vector of overloads

$$M_k = (M_{1k}, M_{2k}, M_{3k}, \ldots, M_{mk})^T \tag{6}$$

describes the initial pattern of loading in the network on day k and is part of the state vector of the slow dynamics.

The fast dynamics on day k depend on the initial pattern of loading M_k. In particular, heavily loaded lines are more likely to be involved in cascade of events leading to a blackout. Blackout is defined as load shedding or the network solution becoming infeasible. Some cascades may redistribute power in the network but preserve the loads and this outcome is not counted as a blackout. Also on some days there may be no events. The outcome of the fast dynamics on day k is a list of the lines that overloaded or outaged during that day's cascade and whether or not there was a blackout.

2.4. Improving the lines

We first describe the procedure for increasing the transmission line capacity based on which lines were involved in the blackout. Then the ways in which this procedure models improvements to the system capacity are discussed.

After the fast dynamics are finished, the line flow limits for the next day are obtained. If there was no blackout, the line flow limits are unchanged. If there was a blackout then the line limits of those lines that overloaded or outaged during the cascade are increased:

$$F_{j(k+1)}^{max} = \begin{cases} \mu_k F_{jk}^{max} & \text{; blackout and line } j \text{ overload on day } k \\ F_{jk}^{max} & \text{; otherwise} \end{cases} \tag{7}$$

where $\mu_0, \mu_1, \mu_2, \cdots$ are independent, identically distributed bounded continuous random variables in $[\mu_{\min}, \mu_{\max}]$ with mean value $\overline{\mu}$. The mean value $\overline{\mu}$ controls the size of the average increase in the line flow limit. For example, $\overline{\mu} = 1.05$ specifies a 5% average increase in the line flow limit. More generally, $\overline{\mu}$ controls the average rate of improving the transmission system capacity. We assume that

$$1 < \lambda_{\max} < \mu_{\min} \qquad (8)$$

In practice it is customary for utility engineers to make prodigious efforts to avoid blackouts and especially to avoid repeated blackouts with similar causes. These engineering responses to a blackout occur on a range of time scales longer than one day. Responses include repair of damaged equipment, more frequent maintenance, changes in operating policy away from the specific conditions causing the blackout, installing new equipment to increase system capacity, and adjusting or adding system alarms or controls. The responses reduce the probability of events in lines related to the blackout, either by lowering their probabilities directly or by reducing component loading by increasing line capacity or by transferring some of the line loading to other lines. The responses are directed towards the lines involved in causing the blackout. Thus the probability of a similar blackout occurring is reduced, at least until load growth degrades the improvements made.

The modeling in (7) is a crude representation of these responses to the blackout. The response is modeled as happening only on the next day, but the effect of the modeled response does persist until it is effectively cancelled by the slow load increase. In cases in which the response lowers the probability of line failure or transfers loading to other lines the model roughly approximates these responses as an increase in the line capacity. (As detailed in section 3, a line with an increased capacity will have a smaller probability of outage or overload.)

The update of the line flow limits can also be expressed as an update in the fractional overloads, taking into account the daily flow increase (3):

$$M_{j(k+1)} = \begin{cases} \mu_k^{-1}\lambda_k M_{jk} & \text{; blackout and line } j \text{ overload on day } k \\ \lambda_k M_{jk} & \text{; otherwise.} \end{cases} \qquad (9)$$

The generator maximum power limits are assumed to grow with the average load:

$$P_{ik}^{max} = (\overline{\lambda})^{k+1} P_{i0}^{max} \qquad \text{; bus } i \text{ a generator} \qquad (10)$$

The model does not represent generator outages.

3. Fast dynamics of cascading events

The fast dynamics of the cascading events on day k are described by the (lower case) variables

$$\begin{aligned} f_j &= \text{power flowing on line } j \text{ during events on day } k \\ f &= (f_1, f_2, f_3, \ldots, f_m)^T \\ p_i &= \text{power injected at bus } i \text{ during events on day } k \\ p &= (p_1, p_2, p_3, \ldots, p_n)^T \end{aligned}$$

The dependence of these variables on the day k is omitted from the notation for brevity.

The power injection and flow vectors p and f are updated as the cascade proceeds. The fast dynamics are initialized with the initial flows and injections for day k:

$$f = F_k \qquad (11)$$

$$p = P_k \qquad (12)$$

The initial flows and injections f and p do not necessarily satisfy the network constraints.

3.1. Initiating random line outages

Blackouts are often initiated by line outages caused by weather, operator error, or device malfunction or failure such as false line trips or substation fires. This initiating event is modeled as an outage of a line or lines according to an independent probability of failure of each line:

$$\text{Probability}\{\text{line } j \text{ outaged}\} = h^0(M_{jk}) \qquad (13)$$

where h^0 is a positive and non-decreasing function.

3.2. Random outage of overloaded lines

Overloaded lines are sometimes outaged either automatically or by operator action. Alternatively, the overload may be corrected without a line outage. This is modeled as an outage of a overloaded line according to an independent probability of failure of each line:

$$\text{Probability}\{\text{overloaded line } j \text{ outaged}\} = h^1(M_{jk}) \qquad (14)$$

where h^1 is a positive and non-decreasing function.

3.3. Power redispatch

Whenever a line is outaged or overloaded, or a generator exceeds its maximum limit, it is necessary to redispatch the injected powers to satisfy the system constraints. The injected powers include both generators and loads but generation redispatch is much preferred to load shedding. (If it is impossible to satisfy the system constraints, then the network solution is infeasible and this is considered to be a blackout.)

The redispatch is formulated in a conventional way as an optimization to minimize the change in generation or load shed subject to the system constraints [10, 11, 12]. The optimization minimizes the cost function

$$\sum_{generators} |p_i - P_{ik}| + \sum_{loads} 100(p_i - P_{ik}) \qquad (15)$$

subject to overall power balance

$$\sum_{i=1}^{n} (p_i - P_{ik}) = 0 \qquad (16)$$

and the line flow limits

$$-F_{jk}^{max} \le f_j \le F_{jk}^{max} \qquad j = 1, 2, \cdots, m \quad (17)$$

$$\text{where} \qquad f = Ap \qquad (18)$$

and the requirement that load shedding be positive and less than the total load

$$P_{ik} \le p_i \le 0 \qquad ;\text{ bus } i \text{ a load} \qquad (19)$$

and the generator limits

$$0 \le p_i \le P_{ik}^{max} \qquad ;\text{ bus } i \text{ a generator} \qquad (20)$$

Note the hundredfold larger weight on load shedding so that load shedding is much more expensive than generation shift [11]. The A matrix used in (18) must incorporate the effects of any line outages. A slightly more detailed implementation of this optimization as a linear program is presented in Appendix A.

The optimization is a simple model of how operators might redispatch power in response to an overload or an outage. Blackouts can also include events which cascade without intervention from the operator; in these cases, the optimization gives a solution compatible with the system constraints with less assurance that it is representative of the actual redispatch.

3.4. Iteration

The cascading events are computed by the following iteration.

1. Initialize the flows and injections according to (11) and (12).

2. Determine the initiating event line outages (if any) according to (13).

3. If any limits are not satisfied or if a line has been outaged in the previous step, redispatch the power injections according to the optimization in section 3.3. If the optimization is infeasible, stop the iteration. Produce a list of lines that were overloaded during the optimization.

4. For each line that was overloaded in step 3, determine whether it is outaged according to (14).

5. If lines were outaged in step 4, then go to step 3. If no lines were outaged in step 4, then stop the iteration.

Model blackout is defined as the occurrence of load shedding during the iteration or the optimization being infeasible. The output of the iteration is a list of the lines that overloaded or outaged during the iteration and whether or not there was a blackout.

The iteration represents cascading events in two ways: Line outages can cascade as the iteration steps proceed. Also line overloads can cascade during the solution of the optimization in step 3.

The cascade modeling only seeks to produce a list of lines that could plausibly be involved in cascading events which lead to a blackout. In particular, the cascade is consistent with the network topology and constraints. However, the modeling does not seek to reproduce any details of the cascade.

4. Analogy with the sandpile

We briefly indicate the roughly analogous structure and effects in a sandpile model that shows self-organized criticality [2, 8]. This rough analogy was used to guide the construction of the blackout model.

Consider a large, idealized sand pile that has grains of sand added at a continuously varying location. When the local maximum gradient gets too large, sand at that location is more likely to topple. Events are the toppling of sand and cascading events are avalanches. The system state is a vector of maximum gradients at all the locations in the sand pile. The driving force is the addition of sand, which tends to increase the maximum gradient, and the relaxing force is gravity, which topples the sand and reduces the maximum gradient. These opposing forces cause the system to converge to a dynamic equilibrium in which the average sandpile gradient is below the angle of repose. In the dynamic equilibrium, avalanches of all sizes occur and there are long time correlations between avalanches.

The analogy between the main quantities in the sand pile and the power system is summarized in Table 1. There are also some distinctions between the two systems. In the sandpile, the avalanches are coincident with the relaxation

Table 1. Analogy between power system and sandpile

	power system	sandpile
system state	fractional overloads M	gradient profile
driving force	load increase λ	addition of sand
relaxing force	line improvements μ	gravity
event	line limit or outage	sand topples
cascade	cascading lines	avalanche

of high gradients. In the power system, each blackout occurs on fast time scale (less than one day), but the knowledge of which lines caused the blackout determines which line capacities are improved after the blackout. The blackout model has inherent inhomogeneity due not only to the inhomogeneity of the transmission line network, but also due to the inhomogeneous distribution of loads and generators. Many sandpile models have much greater homogeneity.

5. Results

5.1. IEEE 73 bus RTS-96

The IEEE Three Area Reliability Test System–1996 is a benchmark system for bulk power reliability studies described in detail in [7]. The system has 73 buses and 108 lines and consists of 3 identical areas connected by a few tie lines.

To extract the data needed for the DC load flow computation, reactive power injections and line capacitances and resistances are neglected. The generation at buses $113, 213, 313$ is adjusted to 136 MW to give overall power balance. The initial MW line flow limits F_{j0}^{max} are approximated by the long-time emergency MVA ratings (24 hours) from [7]. Parallel lines are combined into one line and the optional DC link is not included.

Table 2. Model parameters

	value	comment
$\overline{\lambda}$	1.00005	mean load daily growth factor
λ	$r\overline{\lambda}$	r uniformly distributed in $[1/1.4, 1.4]$
μ	1.005	line improvement factor
h^0	0.001	probability of line outage
h^1	0.3	probability of overloaded line outage

The main model parameters are shown in Table 2. The average daily load growth factor $\overline{\lambda} = 1.00005$ corresponds to an annual load growth of approximately 2%. (The 2% annual growth rate is based on the growth of US electric utility production from 2500 billion kWh in 1985 to 3000 billion kWh in 1995 according to the graph on page 3 of [3].) The line improvement factor μ (generally a random variable) is a constant and the probabilities h^0 and h^1 (generally functions of fractional overload) are also constants. The implementation of the model is described in more detail in [6].

To examine the model behavior, a time series was generated using the model on the IEEE RTS-96 system. The time series has 50,000 days (about 137 years) and 3926 blackouts (about 29 per year).

5.2. Power served

Since the load increase is fixed, the power served depends on the line improvement factor μ. If μ is low, lines are more often stressed, there are a larger number of blackouts and the average power served is lower. Figure 1 shows the power served for two values of μ. The power served grows exponentially to match the load increase, but in the case of the lower value of μ (upper curve in Figure 1) the power served has a higher number of larger negative spikes corresponding to the power shed.

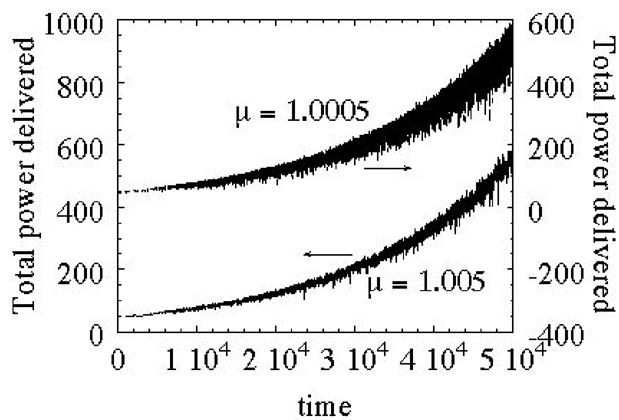

Figure 1. Power served for two values of μ.

If the line improvement factor μ is a constant, then the line improvements must be applied sufficiently frequently so that the network capacity can follow the increasing load. In the model, line improvements arise from the blackouts and the overall system improvement depends on the number and extent of the blackouts. There is a self regulating process by which the system produces a distribution of the number and size of blackouts which gives the average rate of system improvement required by the load increase. Through this process the system reaches a kind of equilibrium in a dynamical or statistical sense. This dynamic equilibrium exists despite the secular evolution of the supply and demand. The dynamic equilibrium is better illustrated with the evolution of quantities such as the fractional line overloads.

5.3. Fractional line overloads

The fractional overloads M_{jk} form part of the state vector of the slow dynamics and describe the daily pattern of loading in the system. Since there are 108 lines, the fractional line overloads evolve in the 108 dimensional Euclidean space R^{108}. M_{jk} of line j on day k can be averaged over the lines to give $\langle M_{jk}\rangle_j$ or averaged over time to give $\langle M_{jk}\rangle_k$. Both are useful as they demonstrate different aspects of the system dynamics.

The time evolution of the line average $\langle M_{jk}\rangle_j$ is shown in Figure 2. Figure 2 shows a dynamic equilibrium in the line averaged fractional overloads which is reached after about 20000 days (about 55 years).

There are fluctuations around the mean value which get larger as $\langle M_{jk}\rangle_j$ generally increases to equilibrium. The fluctuations get larger when the lines are generally closer to the system capacity because in the more highly stressed network there are more opportunities for cascades propagating further and hence larger blackouts.

A slow evolution to the equilibrium and increased fluctuations near the system capacity can also be observed in the evolution of the number of instantaneous topplings in running sandpile models [9].

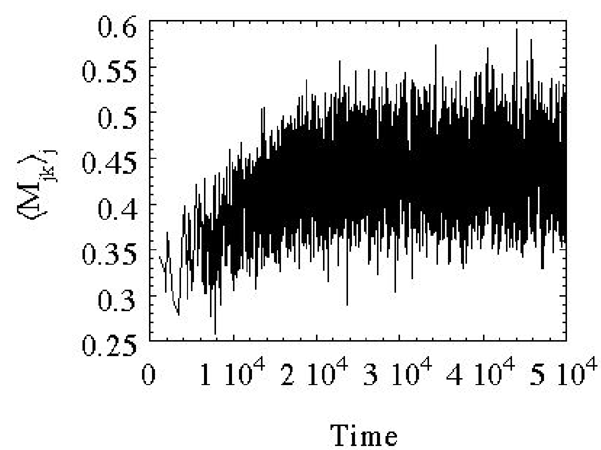

Figure 2. Time evolution of line averaged fractional overloads $\langle M_{jk}\rangle_j$

Once the system has reached dynamic equilibrium we can compute the time averaged fractional overloads $\langle M_{jk}\rangle_k$ for each line. Figure 3 shows $\langle M_{jk}\rangle_k$ for each line and compares the values for three different calculations using line improvement factors μ that vary by two orders of magnitude. The differences in μ must cause the system equilibrium dynamics to differ in their detailed evolution. In spite of the large differences in μ, the three curves are remarkably similar and the distribution of $\langle M_{jk}\rangle_k$ over the lines appears robust. Moreover, our experience is that the distribution of $\langle M_{jk}\rangle_k$ is insensitive to how the dynamic equilibrium was arrived at.

The particular form of the distribution of $\langle M_{jk}\rangle_k$ over the lines depends on the structure of the network. In particular, the distribution of $\langle M_{jk}\rangle_k$ depends on the distribution of generators and loads as well as the line configuration. Thus the distribution of $\langle M_{jk}\rangle_k$ seems to contain interesting information intrinsic to the power system.

In the running sandpile, the quantity analogous to $\langle M_{jk}\rangle_k$ is the time averaged local sandpile gradient. The rate at which the sandpile is driven can be varied by varying the rate of addition of sand grains. The sandpile self organizes to a dynamic equilibrium in which the averaged gradient has a profile which depends on the local coupling and the distribution of sources but is robust to perturbations and the rate of addition of sand grains.

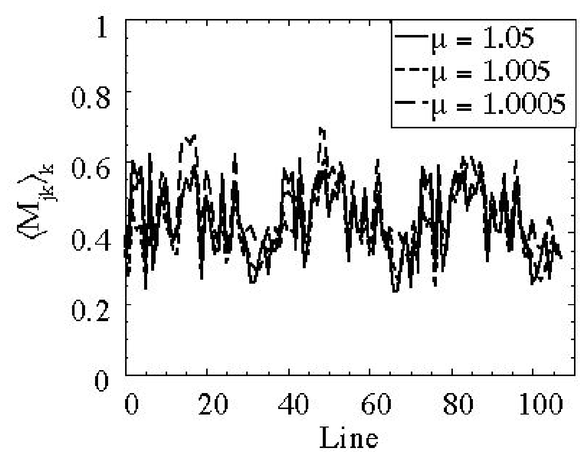

Figure 3. Line distribution of time averaged fractional overloads $\langle M_{jk}\rangle_k$ in steady state.

5.4. Blackouts

After the initial transient, the dynamical evolution of the system leads to a series of cascading events that are blackouts when there is load shedding. Figure 4 shows the number of line outages per blackout as a function of time. The number of line outages per blackout is one measure of blackout size. As with the fractional overload evolution, this time series also reaches an apparent dynamical equilibrium which includes events of all different sizes, once again characteristic of this type of complex system dynamics.

Since we have plotted the outages for a long period of time, Figure 4 gives the impression of a continual blackouts. On the contrary, blackouts are intermittent and in fact happen rather sparsely as illustrated by Figure 5 which shows only a few blackouts in a period of 100 days.

It is important to point out that after each blackout the lines responsible are "improved", thereby removing the immediate cause of the blackout. Nevertheless, the system will still have another blackout caused by another weakness at some time. It is characteristic of these systems that while individual causes can be fixed or avoided, there will always be some system weakness or trigger which will start a cascade that will lead to another blackout.

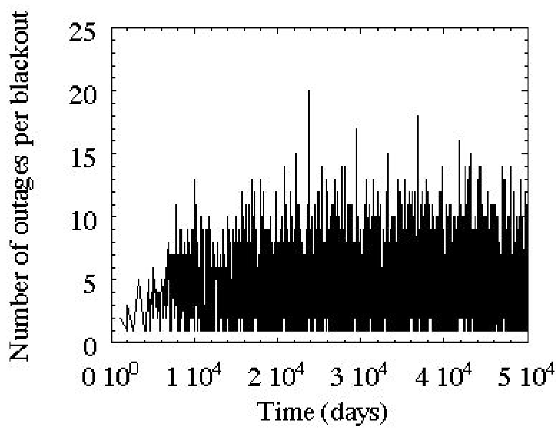

Figure 4. Blackout size time series

6. Comments on Markov aspects

It is convenient to define the load growth factor for day k relative to day 0:

$$\Lambda_k = \prod_{\ell=1}^{k} \lambda_\ell \tag{21}$$

Equations (3) and (4) for the initial daily power injections and line flows become

$$P_k = \Lambda_k P_0 \tag{22}$$
$$F_k = \Lambda_k F_0 \tag{23}$$

The slow dynamics can be characterized by the evolution of the vector of flow limits F_k^{max} and Λ_k. Since Λ_k specifies the initial daily flows F_k via (23), an equivalent characterization is the evolution of the vector of fractional overloads

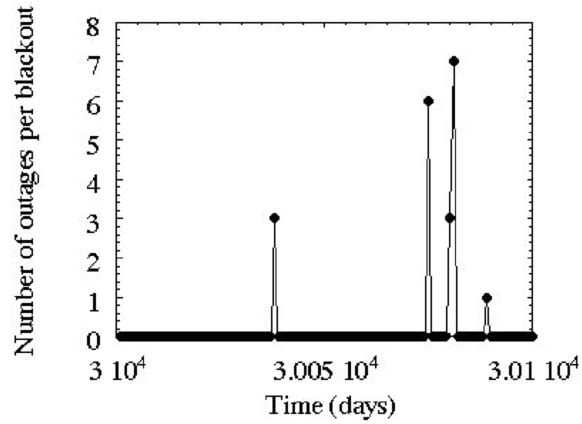

Figure 5. Blackout size time series for 100 days only

M_k and Λ_k. It is better to work with the fractional overloads because the fractional overloads seem to remain bounded. Thus the state space is R^{m+1}, where m is the number of transmission lines.

Given M_k and Λ_k on day k, one can obtain the initial power injections P_k and flows F_k using (22) and (23). Then one can obtain F_k^{max} from M_k and F_k using (5). The cascading events on day k depend only on M_k, P_k, F_k^{max}, and F_k. (Note that the probabilities h^0 and h^1 depend only on M_k.) The next day's initial margins M_{k+1} are obtained from the outcome of the cascade on day k via the independent random variable μ_k (see (9)). Λ_{k+1} is simply given by $\Lambda_{k+1} = \lambda_{k+1}\Lambda_k$, where λ_{k+1} is an independent random variable. There is no dependence on days previous to day k. Therefore $(M_1, \Lambda_1), (M_2, \Lambda_2), (M_3, \Lambda_3), \cdots$ is a discrete time, continuous state Markov process. This Markov property of the evolution of the slow dynamics may be useful in future analysis of the model.

6.1. A simple case

Some of the features of the model can be observed in a very simple case. Consider a single transmission line with a generator at one end and a load at the other end. We can focus on the line overloads only by choosing $h^0 = 0$ and

$$h^1(M_{jk}) = \begin{cases} 0 & ; M_{jk} \le 1 \\ 1 & ; M_{jk} > 1 \end{cases} \tag{24}$$

so that line outages happen when the line is overloaded. Overloading of the single line always implies load shedding and blackout. That is, the fast dynamics reduce to blackout whenever $M_k > 1$.

It is convenient to define

$$x_k = \log M_k \quad (25)$$

$$a_k = \log \lambda_k \quad (26)$$

$$b_k = \log \mu_k \quad (27)$$

We also write $a_{\min} = \log \lambda_{\min}$, $a_{\max} = \log \lambda_{\max}$, and $b_{\max} = \log \mu_{\max}$. Then the slow dynamics (9) with $x_k \in \mathrm{R}$ are

$$x_{k+1} = \begin{cases} x_k + a_k & ; x_k < 0 \\ x_k + a_k - b_k & ; x_k \geq 0 \end{cases} \quad (28)$$

Note that (8) implies that $a_k - b_k < 0$. There is a globally attracting, positively invariant region $(-\infty, a_{\max})$. If we additionally assume that $a_{\min} \geq 0$ (load never decreases), then there is a globally attracting, positively invariant region $[a_{\min} - b_{\max}, a_{\max})$.

Now we restrict to the special case in which the random variables λ_k and μ_k are constants. It follows that a_k and b_k are constants and we write $a = a_k$ and $b = b_k$. Then $[a - b, a)$ is a positively invariant region and for initial condition $x_0 \in [a - b, a)$ the solution is

$$x_k = b \left\langle k\frac{a}{b} + \frac{x_0 - a}{b} \right\rangle + a - b \quad (29)$$

where the brackets $\langle \cdot \rangle$ indicate modulo 1. In the generic case of a/b irrational, x_k is almost periodic and the average value of x_k is $a - (b/2)$. This illustrates the approximately cyclic nature of the model when applied to a single line.

One could imagine the behavior of the model on a network as many single lines with complicated couplings due to the interactions between the lines when they are limited or outaged. Very long approximately cyclic patterns are possible in the network dynamics and it would be interesting to investigate the relation of these approximately cyclic patterns to the complex dynamics of interest.

7. Conclusion

We have defined a model which includes the essentials of slow load growth, cascading line outages, and the increases in system capacity caused by the engineering responses to blackouts. Lines fail probabilistically and the consequent redistribution of power flows is calculated using the DC load flow approximation. Cascading line outages leading to blackout are modeled and the lines involved in a blackout are predicted. The engineering response to the blackout is crudely modeled as an increase in line margin for the lines that were involved in the blackout. The model implements the qualitative explanation of self-organized criticality in power system blackouts proposed in [4].

As appropriate for a first study of global dynamics, the model represents the forces conjectured to cause self-organized criticality in a simple fashion. Moreover, the particular forms of the model simplifications are chosen with a view to improving the tractability of simulation and analysis. The slow dynamics of the model is a discrete time Markov process with a continuous state space of high dimension.

To begin our investigation of the model, we have illustrated its characteristics on a lossless version of the 73 bus three area IEEE Reliability Test System-1996. The model converges to a steady state which is the dynamic equilibrium of interest. The time average of the pattern of overloads in the dynamic equilibrium appears to be a robust feature of the dynamic equilibrium. Similarities of the results with those from a self-organized critical running sandpile are noted.

A companion paper [6] studies the model in a variety of artificial power networks with a regular structure. Of particular interest in [6] are the results on a regular 94 bus network with 3 lines connected to each bus. Although this is an artificial network with more regularity than a real power network, three lines incident on each bus is approximately the average for large power networks [1]. The probability density function of the blackout sizes obtained from the model shows good agreement with the corresponding probability density function from a time series of avalanches from a running sandpile model. Since the running sandpile model is a defining example of self-organized criticality, these initial results suggest that the model can produce a result consistent with self-organized criticality.

The model shows promise for probing the complex dynamics of power system blackouts. However, even such a simplified model is complicated and we can not yet claim definitive results from the few cases briefly examined so far. Much more work is needed to use the model to try to understand the complex dynamics of power system blackouts. The first tasks are to study and refine the model in large power networks of a more typical topology and to improve the extent and methods of analysis.

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A. LP formulation in incremental variables

It is convenient to let $\Delta p_i = p_i - P_{ik}$. Also, to implement the absolute value in the generator part of the cost function, let $\Delta p_i = \Delta p_i^+ - \Delta p_i^-$ for bus i a generator. Then the optimization becomes the linear program:

Minimize the cost function

$$\sum_{i \in G} (\Delta p_i^+ - \Delta p_i^-) + \sum_{i \in L} 100 \Delta p_i$$

subject to

$$\sum_{i=1}^{n} \Delta p_i = 0$$

$$-F_{jk}^{max} - F_{jk} \le \Delta f_j \le F_{jk}^{max} - F_{jk} \qquad j = 1, 2, \cdots, m$$

$$0 \le \Delta p_i \le -P_{ik} \qquad \text{; bus } i \text{ a load}$$

$$0 \le \Delta p_i^+ \le P_{ik}^{max} - P_{ik} \qquad \text{; bus } i \text{ a generator}$$

$$-P_{ik} \le \Delta p_i^- \le 0 \qquad \text{; bus } i \text{ a generator}$$

where

$$\Delta f_j = f_j - F_{jk} = A\,\Delta p$$

B. Network equations

This appendix reviews a standard DC load flow formulation for readers outside power systems. All bus voltages phasors are 1.0 per unit in magnitude. Let θ_i be the voltage angle at bus i. Define the n vector Θ of voltage angles, including the zero angle of the reference bus.

Let b_{ij} be the susceptance of the transmission line joining bus i to bus j. Transmission line resistance is neglected. The $n \times n$ matrix B is defined by

$$B_{ii} = \sum_{\text{bus } j \text{ connected to bus } i} b_{ij} \qquad (30)$$

$$B_{ij} = -b_{ij} \qquad (31)$$

The DC load flow equations are

$$P = B\Theta \qquad (32)$$

B has rank $n-1$ because of the constraint that the powers in the vector P sum to zero. Inverting (allowing for the singularity of B and using the zero angle of the reference bus) gives

$$\Theta = XP \qquad (33)$$

The flow on the line connecting bus i to bus j is $b_{ij}(\theta_i - \theta_j)$ and this can be written in matrix form for all the lines as

$$F = N\Theta \qquad (34)$$

Then

$$F = NXP = AP \qquad (35)$$