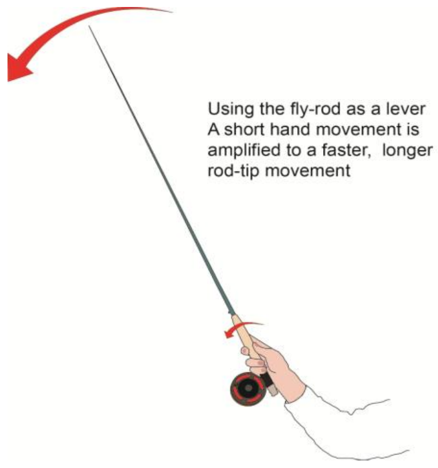
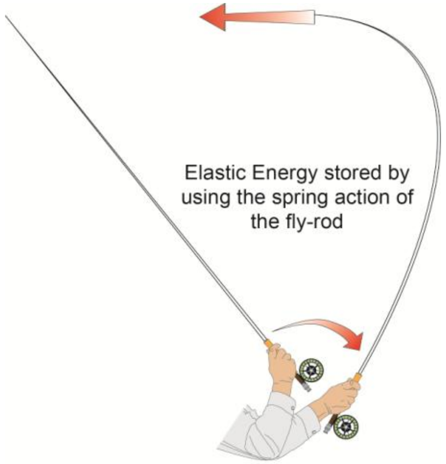
The physics of the overhead fly cast

Introduction

- Primarily a fly rod is a lever that is tapered for practical reasons. Essentially, it is a simple speed amplifier, using rotation in a limited space but at a cost; kinetic energy has to be put into the lever, which is then wasted during the stop at the end of the cast.
- It is also a smart spring, which flattens the tip trajectory to provide an efficient loop in the fly-line. But it does more than this. As it deflects, the spring function of the rod stores energy, most of which will be released into the line, at the end of the cast, providing the timing of the caster's motion is correct. Also at the end of the cast, the rod butt will be stopped and the remaining kinetic energy will be in the rod-tip, which creates the loop shape during counter-flex.
- So the "flexible lever" is a friendly, casting tool, that enables the caster to deliver between 20% to 80% of used energy into the fly-line, depending on conditions, and we are going to look at the reasons why this is so.





Thanks to

- All the aficionados of the SL forum and their challenging questions, criticisms, which allowed to improve the model and understanding the physics.
- John Symonds for polishing my English and adding nice drawings to illustrate this presentation.
- The friend of mine who raised the question first in 1981, the late Harry Wilson, founder of Scott fly rods.

Content

- The objective
- The spring & marble model
- The technique used to get the main equations
- What these equations can tell us (without solving)
- The caster’s input
- A particular case (the “self deceleration mechanism”)
- Main messages from modeling
- Conclusions
- Appendix

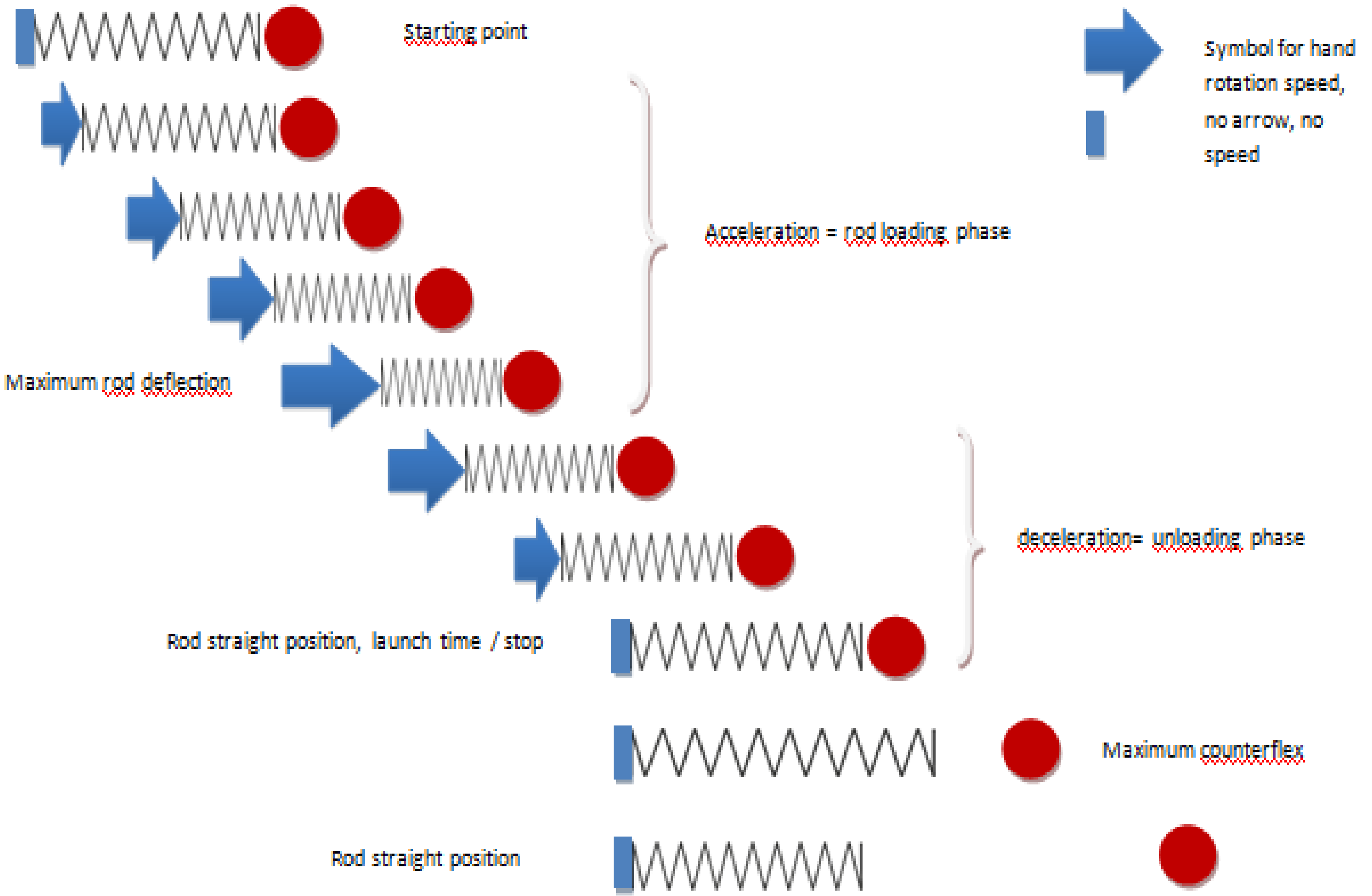
The objective

- To understand the basics of the complex mechanics of the overhead cast.
- When considering a complex mechanism, firstly it is worth simplifying it as much as possible and then later the relevance of the model can be improved as far as it can. Initially, it is important to concentrate on the main issues.
- In which case we assume that air drag and all friction forces are negligible. Such friction and damping forces tend to slow down speeds and reduce amplitudes, but they do not change the main mechanisms involved.
- We also neglect the effects of gravity and use a one-dimensional model consisting essentially of a marble and a spring.

The spring & marble model (1/2)

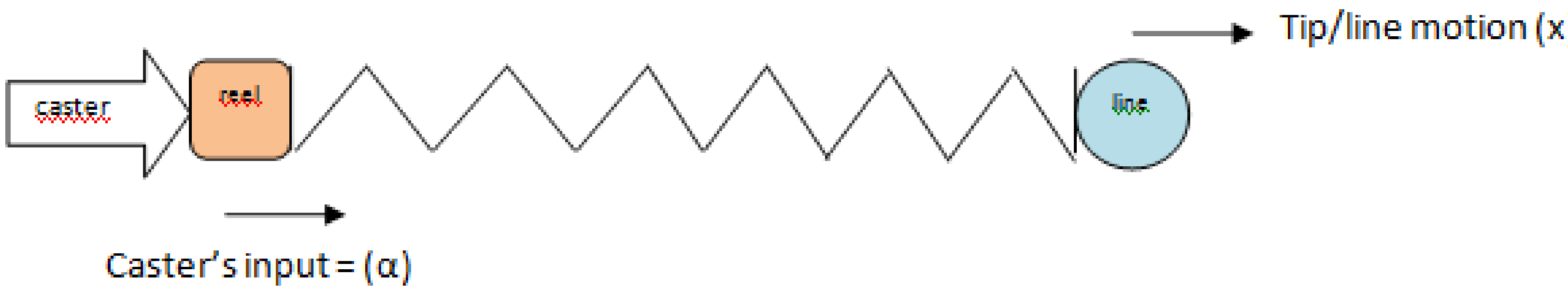
- The rod is represented by a spring and the line by a marble.
- The caster pushes on the spring, which in turn pushes on the marble on a flat surface, along a straight line (simplifying assumptions).
- Thus the lever function of the rod is discounted but it can be reintroduced at a later stage (lever arm effect).
- The caster accelerates and decelerates the spring motion to a stop; the spring is temporarily compressed and launches the marble at some time as it unloads.

A forward cast simulation scheme



α̇

The spring & marble model (2/2)



- This is the scheme that is used to assess the equations of the model; another mass is included to represent a reel, at the butt of the spring.
- The input displacement is alpha (α), its speed is denoted with a dot ($\dot{\alpha}$) and its acceleration with a double dot ($\ddot{\alpha}$).
- The output is denoted x, its speed is denoted with a dot ($\dot{x}$) and its acceleration with a double dot ($\ddot{x}$).
- Initially alpha and x are set at zero.

The technique used to get the main equations (1/3)

- Since we have two parameters (α and x) it is appropriate to use the Lagrangian equations to solve the problem. The specific conditions needed to use this technique are met (degrees of freedom, complete description of the system).
- The Lagrangian is the difference between the kinetic energy of the system and its potential energy (corresponding to the spring energy here).
- This mathematical technique is a very smart way to apply Newton's Laws for mechanical systems.
- The details of the Lagrangian equations are given in appendix.

Note: Joseph Louis Lagrange was a mathematician who put the basis of analytical mechanics and differential equations. Lagrangian mechanics are of great help when Newton's classical approach is no more convenient, such as problems with multiple degrees of freedom.

The technique to get the main equations (2/3)

- Notations:
  - Reel mass = m
  - Spring stiffness = k, constant (linear spring)
  - Spring mass = µ
  - Line mass (marble) = M
- Energies for the Lagrangian:
  - Kinetic (for all masses under motion, including the reel): the classical formulation is 1/2 mass * velocity^2

$$W_{kinetic} = \frac{1}{2} m \dot{\alpha}^2 + \frac{1}{2} M \dot{x}^2 + \ spring\, KE$$

  - Elastic (spring under compression/extension): the classical formulation is ½ stiffness * deflection^2 .

$$W_{elastic} = \frac{1}{2} k (x - \alpha)^2$$

- Lagrangian (L): $L = W_{kinetic} - W_{elastic}$

The technique used to get the main equations (3/3)

- One difficulty is to properly represent the kinetic energy of a moving spring. Most of the time a spring is considered to have no mass and is fitted with an equivalent mass at its tip to represent its behaviour under vibration. This equivalent mass is equal to the third of its actual mass. This is represented by the name “simple spring” here, whilst we call the actual model of the spring we are using with the Lagrangian equations a “true spring”.
  - Simple spring: $$\boldsymbol{spring\,KE} = \frac{\mathbf{1}}{\mathbf{2}}\left(\frac{\mu}{\mathbf{3}}\dot{x}^2\right)$$
  - True spring: $$\boldsymbol{spring\,KE} = \frac{\mathbf{1}}{\mathbf{2}}\left(\frac{\mu}{\mathbf{3}}\dot{\alpha}^2 + \frac{\mu}{\mathbf{3}}\dot{x}\dot{\alpha} + \frac{\mu}{\mathbf{3}}\dot{x}^2\right)$$

- In the appendix we describe the method to clarify this difficulty, and find the final result for energies. There are three components for the mass of the spring: the equivalent mass at the rod-tip, the equivalent mass at butt, and something we call the “equivalent transfer mass”. In the particular case of a spring, they are all equal.

What the equations can tell us (without solving them)

- There are two equations, one corresponding to each variable (α and x):
  - The equation of the motion of the marble at tip(x), from which we can get important information.
  - The equation giving the force needed to cast, which comes from the Lagrangian equation for α.
- Solving the equations can be achieved through a numerical process, but it is not necessary to solve them to capture some important mechanism.

Tip motion equation (1/2)

- Firstly, considering the motion equation: the first line represents the equation straight from the Lagrangian technique application; the second line is the same equation where the acceleration of the tip ($\ddot{x}$) of the spring (or the marble as long as it has not been launched) is expressed in the other terms.

$$\left(M + \frac{\mu}{3}\right)\ddot{x} + \frac{\mu}{6}\ddot{\alpha} + k(x - \alpha) = 0$$

$$\ddot{x} = -\frac{\frac{\mu}{6}}{\left(M + \frac{\mu}{3}\right)} * \ddot{\alpha} + \frac{k}{\left(M + \frac{\mu}{3}\right)} * (\alpha - x)$$

- The first term is associated with input acceleration ($\ddot{\alpha}>0$), and comes from the equivalent transfer mass in the equations. It is negative (minus sign) and consequently:
  - Initially, accelerating from zero, whilst the spring is not yet compressed (meaning that α-x is negligible), the tip is subjected to negative acceleration, which is why its very first reaction is to move backwards.
  - However, if the rod-butt is decelerated rapidly ($\ddot{\alpha}<0$), the first term becomes highly positive and contributes to boosting the acceleration of the tip. This is a major consideration when casting short line lengths (e.g. leader only), and should convince the caster to decelerate his rod quickly to get this boost.

Tip motion equation (2/2)

- The second important thing you can see is the second term, which tells you that the acceleration of the tip depends on the deflection of the spring (α-x) and on the square of the angular frequency of the mechanical system: the faster the system and the larger the deflection is, the higher the acceleration of the tip and marble is.

$$\frac{k}{\left(M+\frac{\mu}{3}\right)}(\alpha - x) \qquad square\ of\ angular\ frequency = \frac{k}{\left(M+\frac{\mu}{3}\right)}$$

- This shows that there is an incentive to create a good bend into the rod, and that the frequency is the single most important parameter of the mechanical system. Mass and stiffness are involved but only the frequency of the system is important (set M = 0 and you have the fundamental frequency of the spring alone): you can imagine several different mechanical systems of this type having the same frequency:

$$frequency = \frac{1}{2\pi}\sqrt{\frac{k}{\left(M+\frac{\mu}{3}\right)}}$$

Butt motion equation (1/2)

- We can make the same analysis, starting from the second Lagrangian equation (first line), and express the butt acceleration equation ($\ddot{\alpha}$) as a function of the other terms:

$$\left(m+\frac{\mu}{3}\right)\ddot{\alpha}+\frac{\mu}{6}\ddot{x}-k(x-\alpha)=F_{\alpha}$$

$$\ddot{\alpha}=-\frac{\frac{\mu}{6}}{\left(m+\frac{\mu}{3}\right)}*\ddot{x}-\frac{k}{\left(m+\frac{\mu}{3}\right)}*(\alpha-x)+\frac{F_{\alpha}}{m+\frac{\mu}{3}}$$

- Since we know already that the tip acceleration ($\ddot{x}$) remains positive as long as the rod has come to its straight position (RSP), and that α-x is positive up to that point too, then you realize that two parameters already exist to decelerate the rod ($\ddot{\alpha}<0$).

Butt motion equation (2/2)

- The term in $\ddot{x}$ comes from the transfer equivalent mass (same as for the tip motion), and the smaller the reel mass and/or the butt equivalent mass are, the higher the deceleration of the butt is.
- The second term relies to spring deflection and is always negative as long as the rod has not reached RSP. So the larger the maximum deflexion is, the larger the deceleration effect on the rod butt.
- The third term is the caster's input force, and then you can speculate that if the two other terms are large enough, then there might not be a need for an extra deceleration due to the caster himself (meaning $F_\alpha < 0$). Any extra mass at butt (reel) is diminishing the intensity of the caster's deceleration (F_α) and should be avoided, as far as possible.

Force involved in moving the spring and all masses

- This is given by the second Lagrangian equation, but we can also replace it with a combination of both equations (their sum) and here Newton's law is clearly visible: the force at the butt of the spring is the sum of the forces exerted on the "reel" mass, on the marble and on the spring itself, where it appears as if its mass is concentrated at is centre of mass (its acceleration is the average of the tip and butt accelerations):

$$m\,\ddot{\alpha} + M\,\ddot{x} + \mu\,\frac{(\ddot{\alpha} + \ddot{x})}{2} = F_{\alpha}$$

- If we had chosen to model the spring as having no mass, with its equivalent mass (μ/3) at the tip only, then we would have found that the force applied by the caster would have to be equal to the ones necessary to compress the spring and to move the reel. Ignoring the reel (m=0), then the required force can be deduced, easily, from the deflection of the spring.

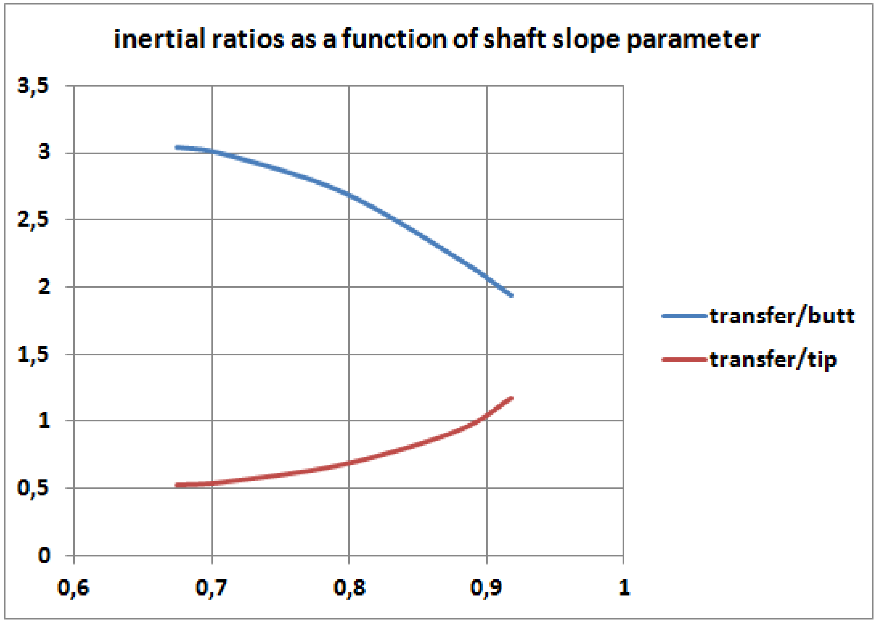
$$(m)\,\ddot{\alpha} - k(x - \alpha) = F_{\alpha}$$

Conclusions about motion equations

- Without solving the equations, we can identify the main parameters involved in the physics of the cast:
  - the natural frequency of the spring & marble system with load (M) at tip,
  - the deflection that the spring will be able to achieve
  - the characteristics of the spring (the components of its mass: the three equivalent masses identified through the analysis of its kinetic energy).
  - and the effect of the caster’s input, especially for the deceleration to a stop (casting style).

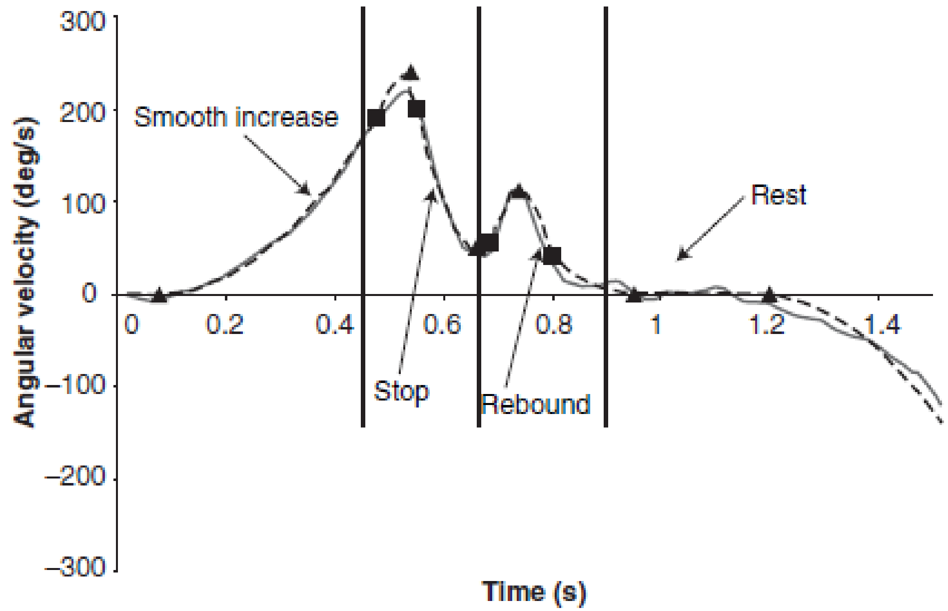
Illustration of two equivalent mass ratios

- If we simplify the conditions (no reel mass, no line mass), then the acceleration / deceleration characteristics linked to the spring are made of two ratio of equivalent masses:
  - The “transfer equivalent mass” / the equivalent mass at tip ($\ddot{x}$), representing tip boost
  - The “transfer equivalent mass” / the equivalent mass at butt ($\ddot{\alpha}$), representing self deceleration
- In our imperfect world, we cannot optimize both at the same time, and here is an illustration for simple shafts of constant slope (fly rods correspond to 0.75 to 0.85 in the abscissa). In other words tip action rods are more prone to inertial transfer for the tip and butt action ones are better for butt self deceleration:



The caster's input

- A very practical way to describe the caster's input is to use the speed of rotation. Some devices can record this (e.g. casting analyser®), and they show an almost linear increase followed by a sharp decrease. At the very end a "rebound" is evident, which is due to rod reaction (counter flex) influencing the movement of the caster's arm. The arm is relaxed during this phase in order to dampen the counter flex and possible vibrations.
- For modeling we ignore the rebound and decrease the speed to zero.



From a publication by D. Anderson, N.C. Perkins and B Richards

A particular case, the free deceleration

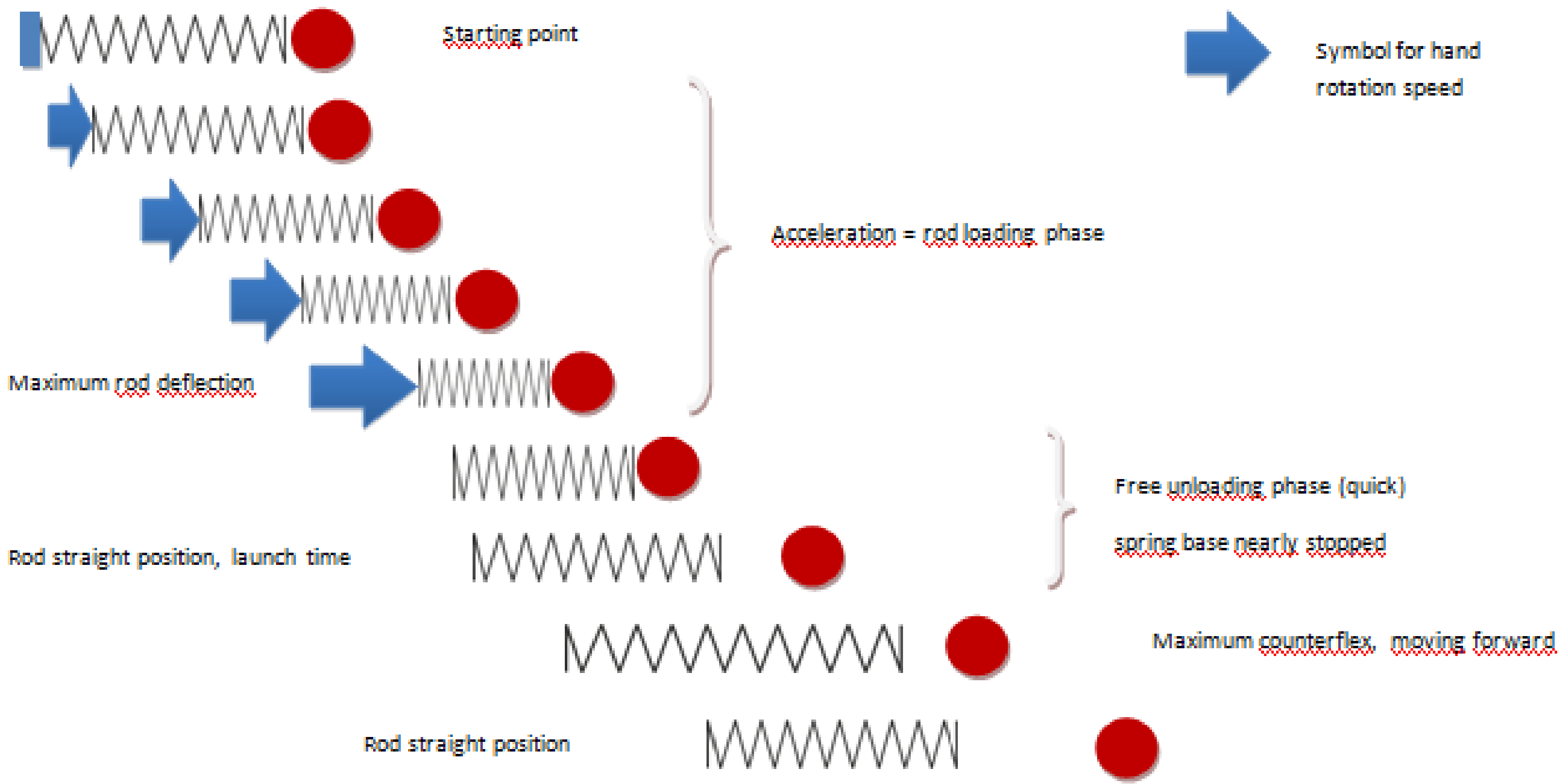
- This applies when the caster ceases to act on the spring at the very end of the acceleration phase, and release grip on the rod butt with the “reel” mass attached, and the marble (line)is pushed by the tip of the spring.
- In this case, from the very moment when the caster ceases to act on the system, these equations become both conservative (no external force any more and so energy is conserved) and both acceleration in α and x become linked together.
- From the model used (M = μ = 10 grams, m≃1.5 gram), we can estimate the relative amplitude between the acceleration of the marble and the deceleration of the butt of the spring, which shows a rapid deceleration of the butt:

$$\ddot{\alpha} \simeq -3\,\ddot{x}$$

- If we where casting an apple with a stick this amplification would be much larger (e.g. 10 instead of 3)

Illustration of the "self deceleration mechanism"

(without reel here)



Butt displacement profile under free deceleration (1/3)

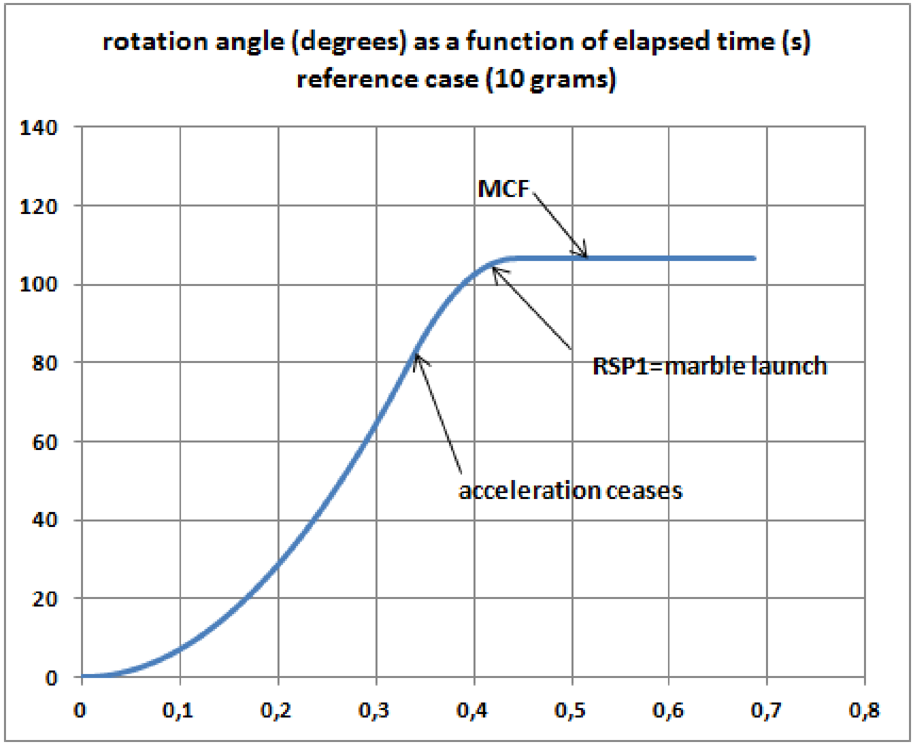
- Below you can find illustrations for various cases:
  - Reference case with a driven deceleration to a complete stop
  - Self deceleration mechanism (SDM), followed by a rebound, then another SDM, etc.

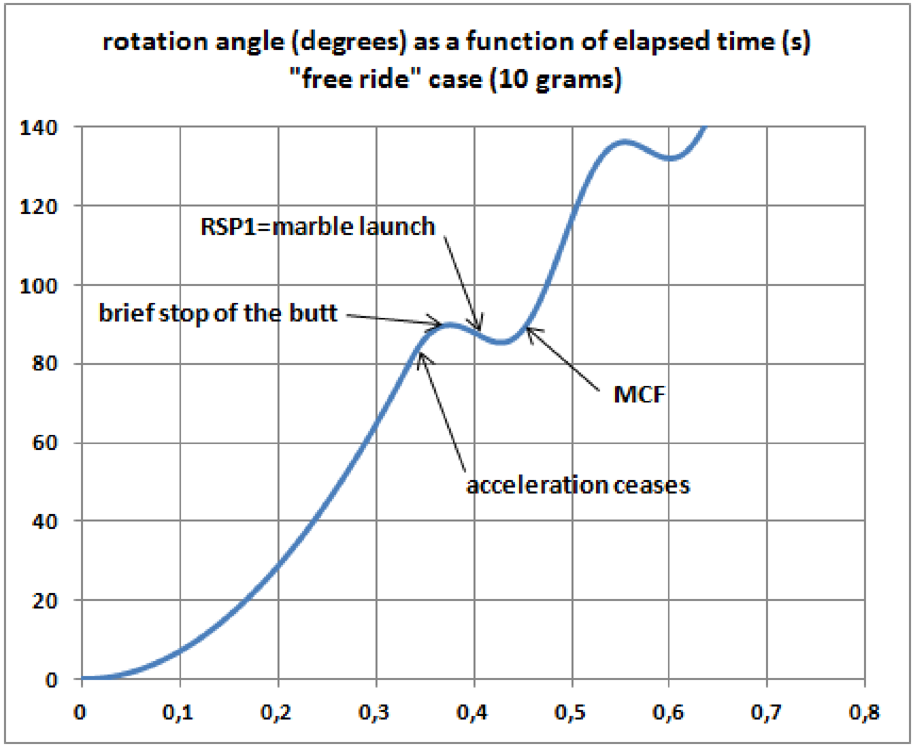
- Notations:
  - RSP = rod straight position (1= first occurrence, 2 = second occurrence, etc.)
  - MCF = maximum counter flex of the spring

- The vertical scale has been adapted to degrees of rotation (of rod butt), which should simplify understanding by casters.

Butt displacement profile under free deceleration (2/4)

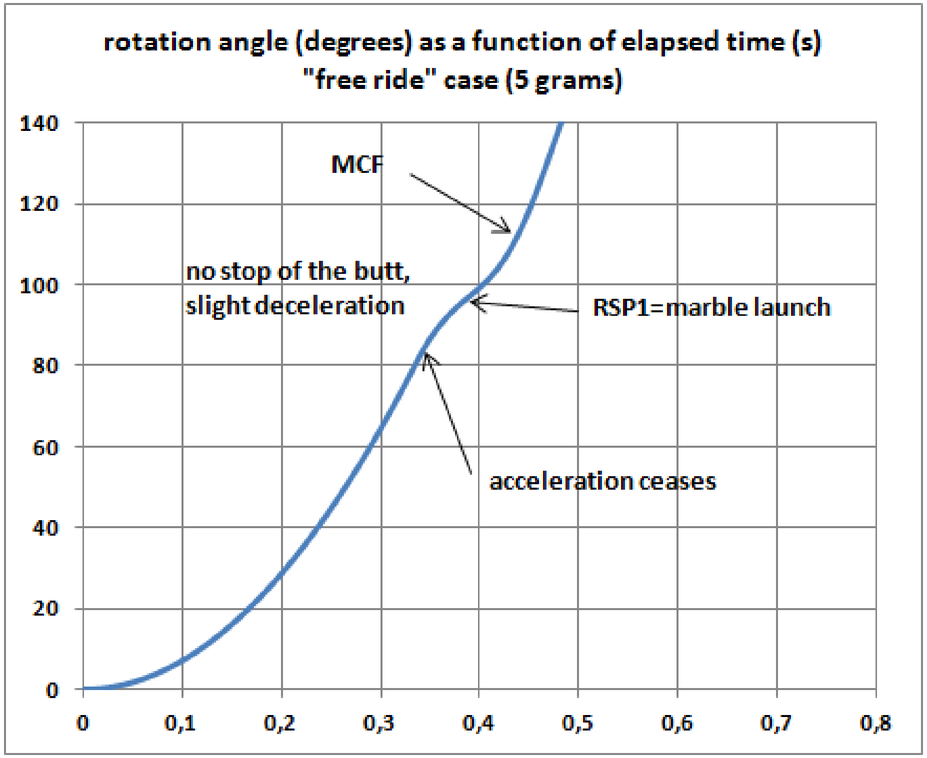
- Diagram on left - the reference cast with a driven deceleration and a complete stop at 107 degrees from start.
- Diagram on right - the SDM mechanism appears, there is a brief stop at 0.37s from start, for an angle of 90 degrees, then the butt moves backwards slightly and forwards again.

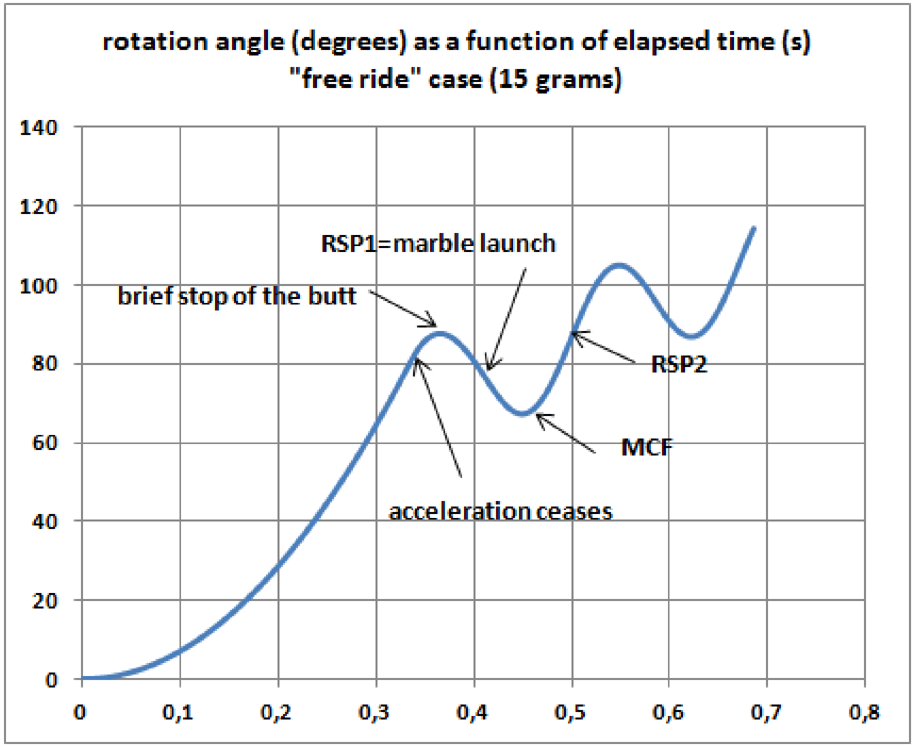




Butt displacement profile under free deceleration (3/4)

- Two other examples where we can see the butt displacement changing, stopping, and even reversing before restarting forward:
  - Light load (marble mass = 5 grams): the SDM is hardly visible
  - Heavy load (marble mass = 15 grams): the SDM is strong and drives the butt of the spring backwards temporarily before a rebound.

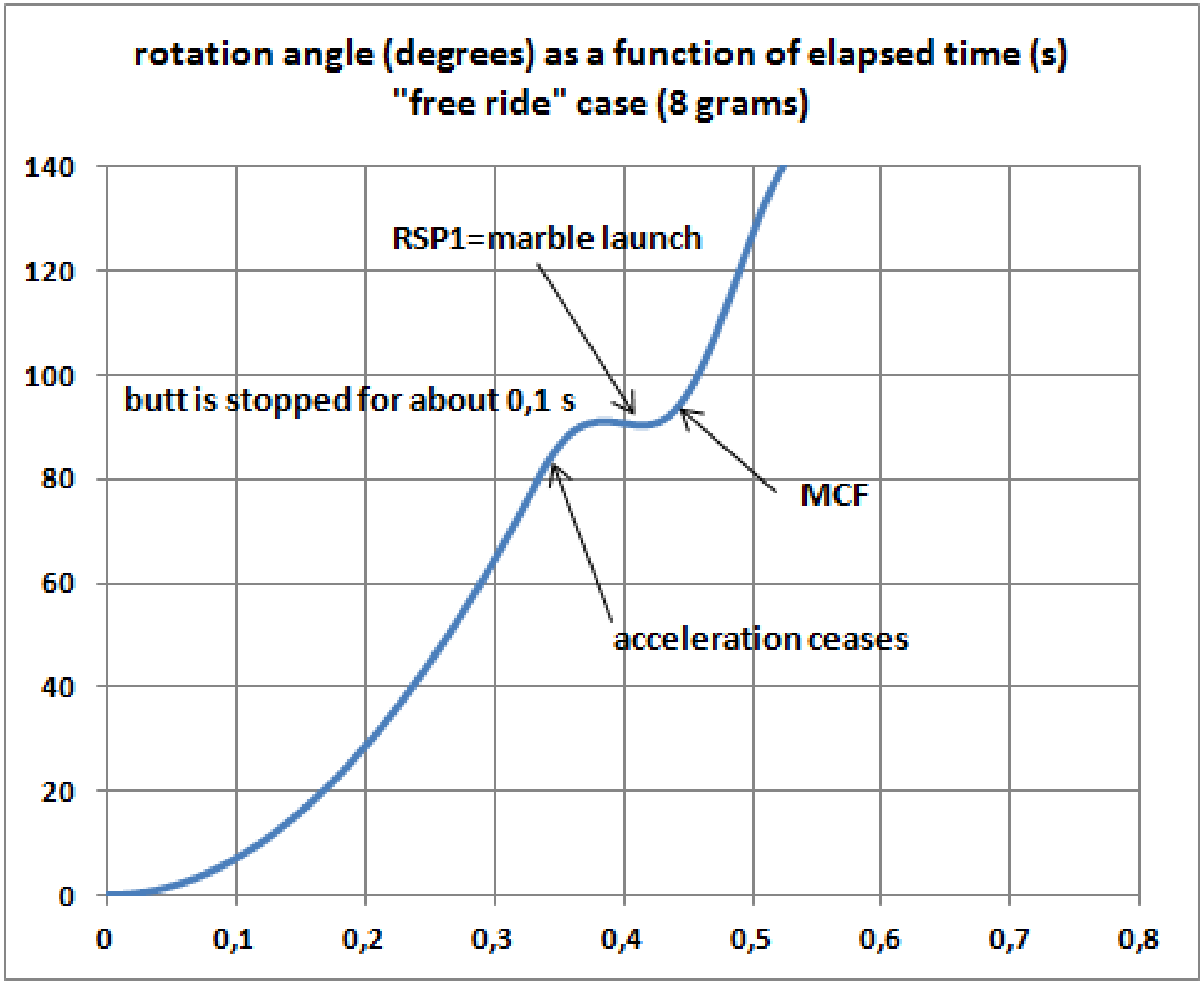




Butt displacement profile under free deceleration (4/4)

- These illustration show that whatever the conditions, the spring is keeps on moving (no energy loss in the assumptions) and that in practice there is always some energy wasted by the caster in order to bring the spring to a complete standstill.
- The conclusion is that the spring has the potential to decelerate its butt, but that the caster has to spend some energy to stop it completely, at the very least by damping the energy due to the counter flex of the rod.
- The SDM mechanism is always within the system but its amplitude is controlled by the mass of the marble and the timing of the cast. This influences considerably the force that the caster has to produce. When the SDM is fully in place, the caster does not need to spend excessive energy to bring the rod to a final stop. If the conditions are not met, then he has to absorb some kinetic energy remaining in the butt of the rod.

"Ideal" SDM condition illustrated

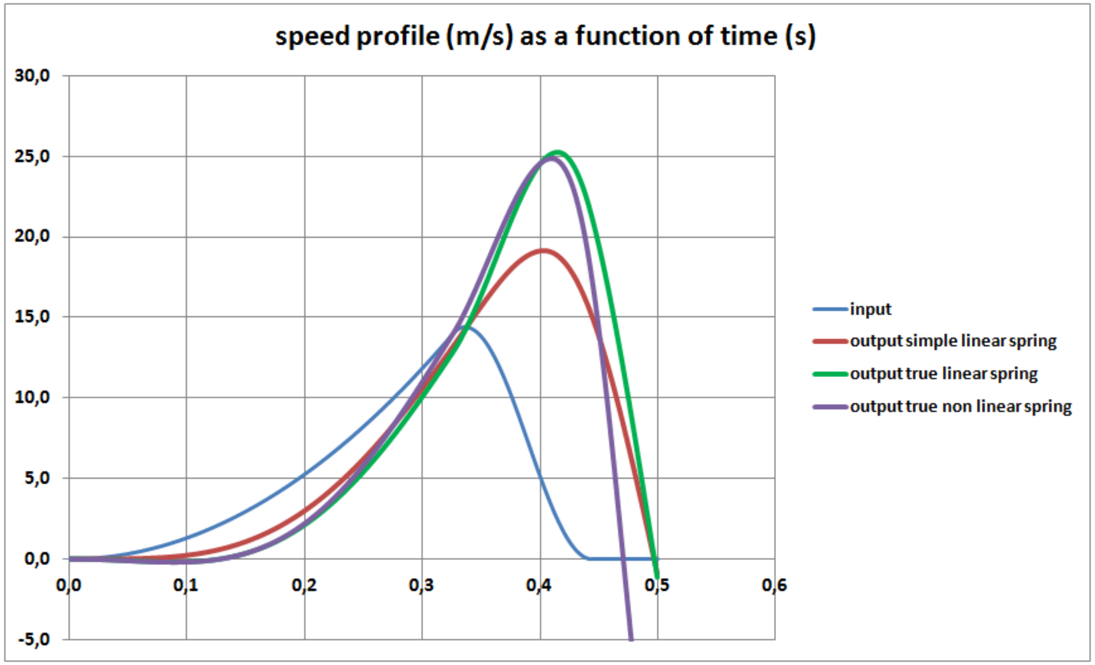


Main results from modelling

- If we solve the equations, then we can fully describe the speed history of the tip in time and test many options by varying inputs:
  - Casting style: change acceleration and deceleration timing and intensity, change the casting arc (the larger it is, the higher will be the speed of the marble at launch)
  - Spring and marble characteristics and load: spring stiffness, spring mass, marble mass (tackle frequency effects)
  - Energy use: caster's input, kinetic energies (spring, "reel" mass, marble) and elastic energy (spring)
- This is an extensive piece of work and here we shall just illustrate a few points, some speed profiles (tuned for realistic figures in terms of fly line speeds), and some deflection profiles. We consider a reference case with a simple spring, and there is already a speed amplification, and then we add the true spring characteristics, and on top of that a non linearity of the stiffness of the spring (hard spring: stiffer and stiffer as its deflection increases).

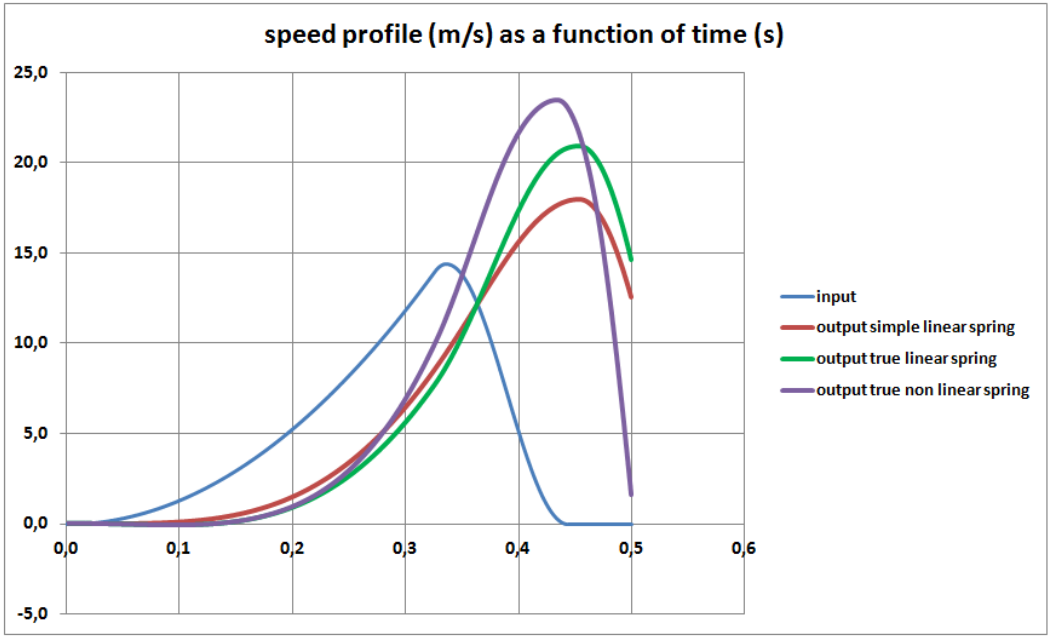
Main messages on speed (1/3)

- A few results can illustrate the influence of the inertial effect due to the equivalent transfer mass and the non linearity of the spring of a fly rod (added on top). The following case corresponds to a “leader only” cast and shows the benefit of the inertial transfer due to the (moderately) sharp deceleration (illustrated by the input speed). The marble is launched when the speed of the tip of the spring (illustrated below) is maximum, in other words at the top of the curves.
- The amplification in speed can be visualized by comparing with the top of the input speed profile.



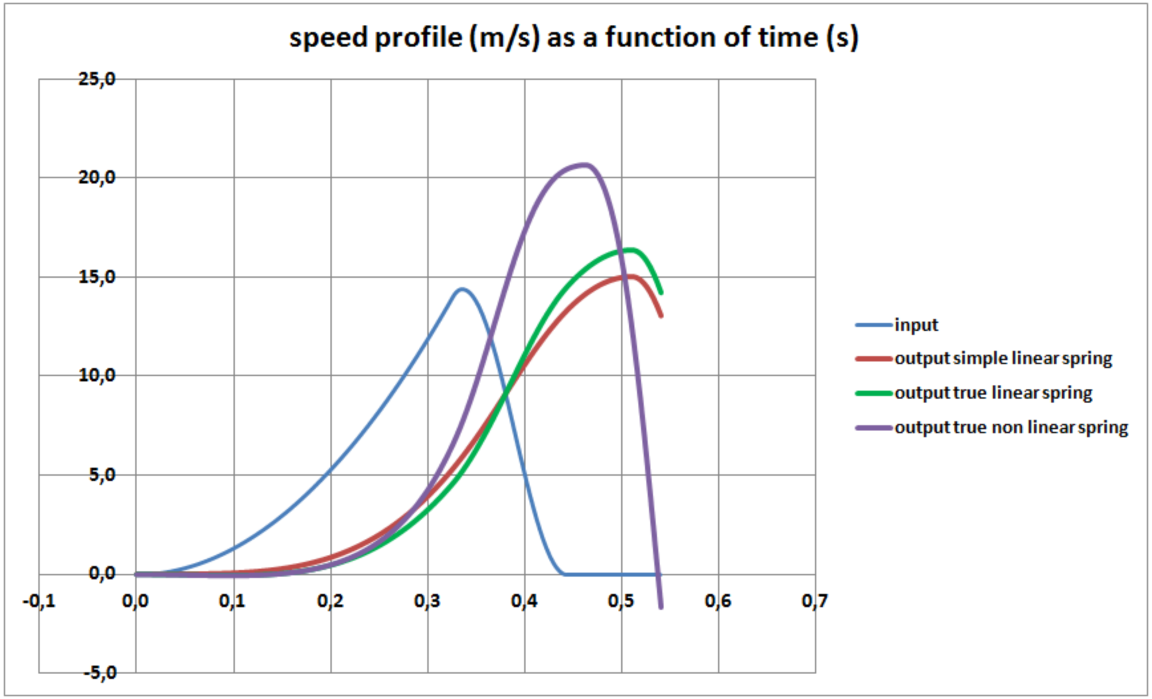
Main messages on speed (2/3)

- Same model with an intermediate load: both inertial effect and non linearity combine with each other to bring a benefit to the caster by comparison the a linear simple spring (which is not the adequate representation of a spring, as discussed earlier): here we are in an intermediate situation where inertial effect and non linearity can gather to bring an extra speed amplification.



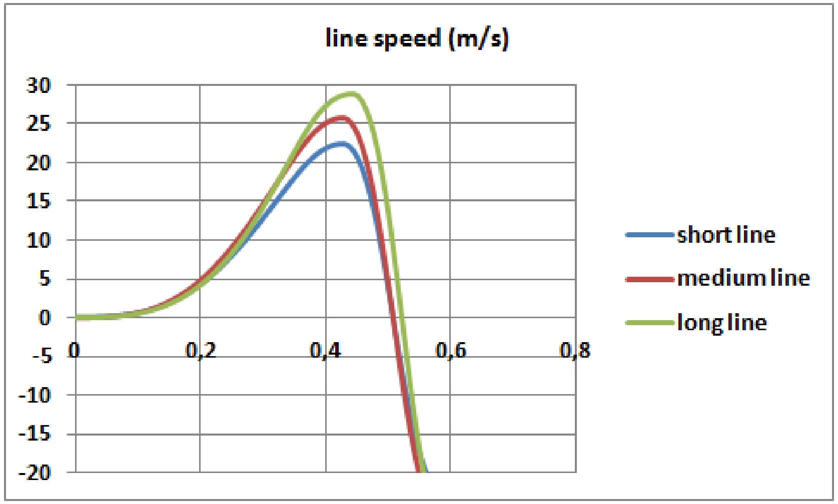
Main messages on speed (3/3)

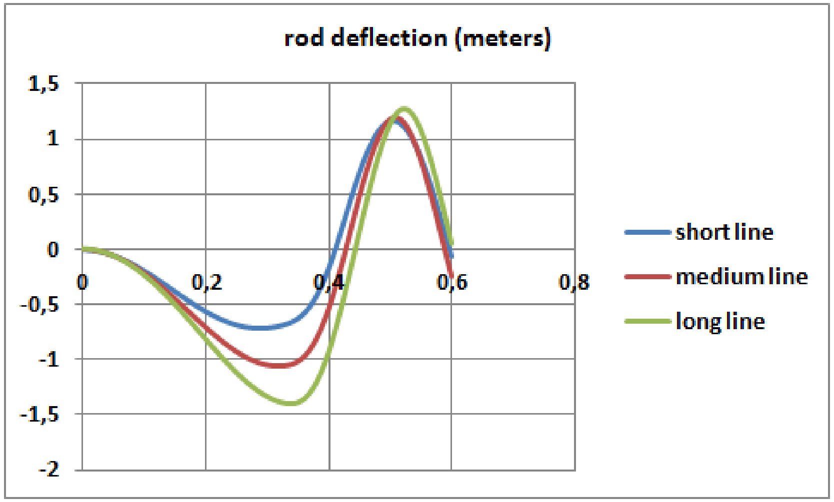
- Now with a heavier load (same model, same assumptions and parameters). The non-linearity has increasingly more influence, whilst the inertial effect is nearly non-existent (it could differ if the casting style in the model is changed). Thus for long casts, the non-linearity of a spring can provide another advantage to the caster.
- It is worth knowing that non-linearity depends very much on rod length: small rods are very non-linear, and the converse applies for long rods.



Main message on “load” (1/2)

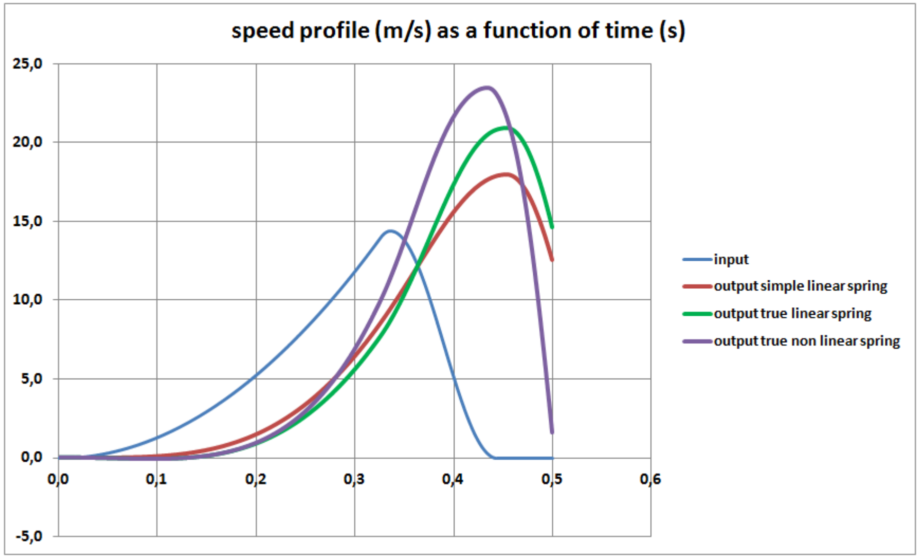
- This is a hot topic since to make the spring “work” properly, the caster has to store some elastic energy in it (see SDM also). However, if load/spring deflection is necessary, marble speed can vary significantly with a similar load. There are many variables involved, particularly the casting style, and there is no simple rule to guide us with regard to “load”. There must be some load but too much will be less efficient.
- First example: as we increase line length and casting arc, both speed and deflection increase, and these are managed by the frequency fit of the cast.

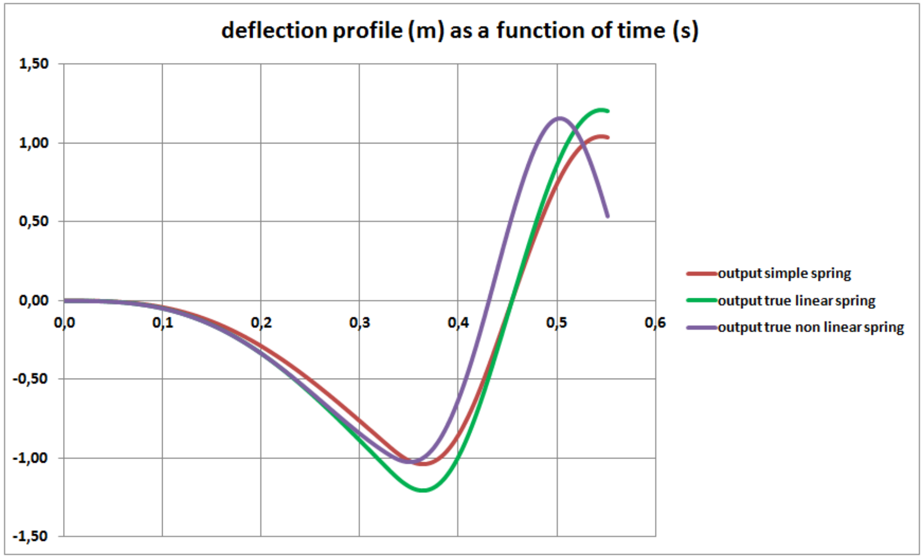




Main message on “load” (2/2)

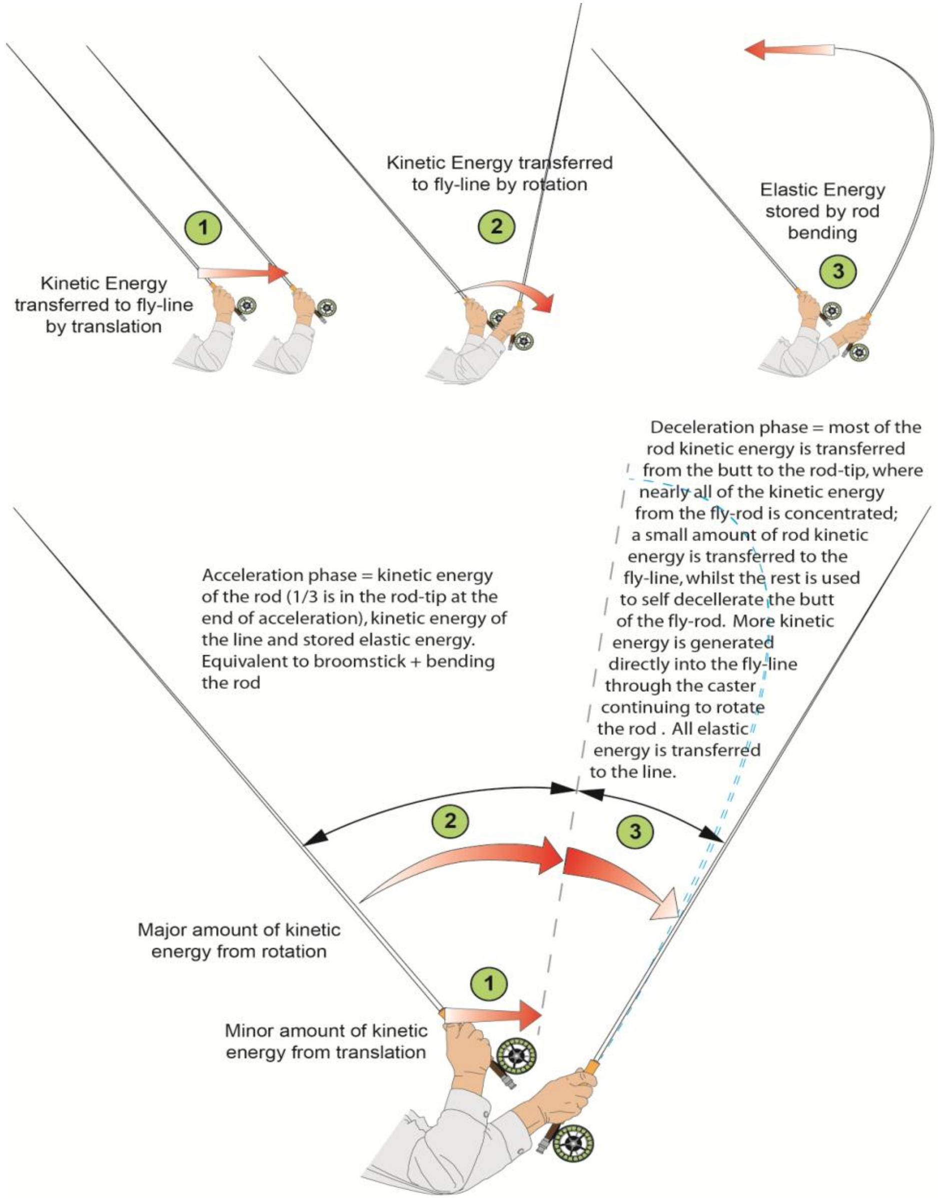
- There is no clear relationship between line speed and rod deflection, this is blurred by other characteristics such as
  - Casting style (e.g. overpowering increases deflection but not line speed)
  - Non-linearity of the rod, as illustrated below (compare the linear rod in green to the non linear rod in purple, for a given cast)





Energy budget

- For simplicity we ignore the reel: accelerating and decelerating it is just a loss of useful energy. The lighter it is, the better.
- Figures can vary significantly; let's concentrate on rod and line, and start by the energy allocation for a fair cast:
  - 75% ends up in the line (kinetic energy), 1/3 coming from elastic storage (rod deflection)
  - 25% ends up in the rod (kinetic energy transferred from elastic energy during counter-flex)
  - 80% of the 25% of the rod energy remains in the tip and is lost at the end of the cast (20% of all energy); the rest is used for butt deceleration and transfer to the line
  - For the line, the remaining 2/3 comes from leverage of the fly-rod during acceleration, and leverage and transfer during the deceleration phase.
- For a poor cast, most of the leverage energy in the line comes from the acceleration phase, very little from the deceleration phase.
  - 40% ends up into the line, 60% into the rod.
  - 30% of line energy comes from elasticity
  - The caster has to produce energy to stop the butt of the rod, increasing the share of rod energy in the allocation.



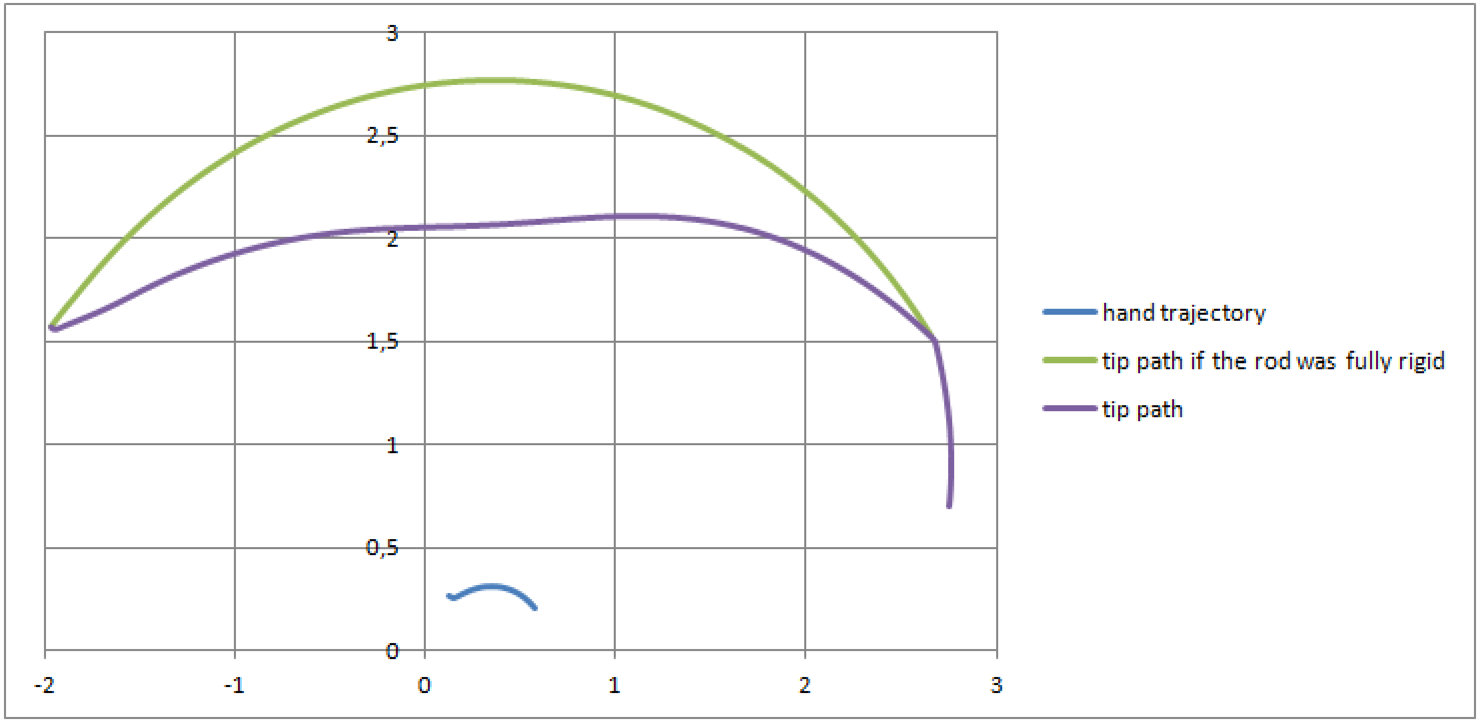
Components of the kinetic energy that is transferred to the fly-line when it is launched

Conclusions

- Modeling can tell us many more things but this review focuses on understanding the behavior of a fly-rod.
- In the overhead fly cast, there are several underlying phenomena which provide an advantage to the caster:
  - The speed amplification effect from the spring which is added to the obvious speed amplification effect of the lever, even though the latter can be moderated by rod deflection
  - The inertial effect coming from a single component of the combined swing weights (equivalent transfer mass / inertia)
  - The non-linearity of the spring (pretty variable with what is called “rod action”).
  - The self-deceleration mechanism, which helps the caster to decelerate and stop the butt of the rod and is mainly linked to the mass and timing of the cast.
- Things vary in amplitude with the “load” of the rod (its deflection due to swing weight, line inertia and stiffness) but this provides a remarkable tool that is able to cast from short to long distance (up to a point), with the help of several underlying mechanisms that combine and/or interact with each other.
- A fly rod is an incredibly smart and complex tool indeed.

Next generation ?

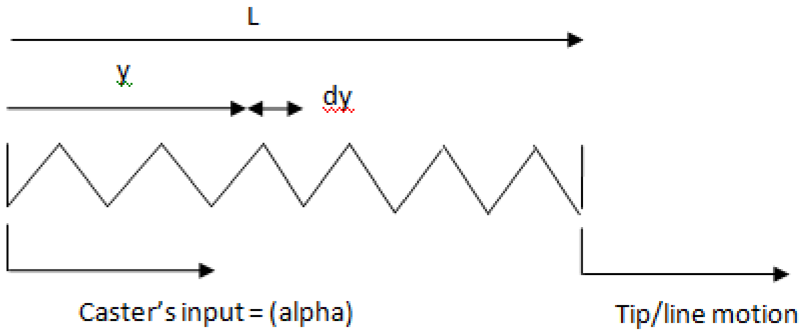
Below you can find an illustration of the tip path of a fly rod for a 2D model (the 0,0 point corresponds to the shoulder joint, scales are in meters). This is a new model under development, and let's hope we can come closer to reality when speaking about the behaviour of a fly rod. The main interest of this 2D model is to understand the role of the bending (shorter lever arm) on rod performance.



Appendix content

- The kinetic energy of a moving spring
- Lagrangian equations
- Conservative case (SDM)
- Adaptation of some equations to fly rods:
  - Scaling masses for the spring & marble model
  - Model adaptation to real rods and shortcuts
  - Angular parameters
  - Self deceleration equation for a rod

The kinetic energy of the moving spring



$$spring\ KE = \frac{1}{2} * \frac{\mu}{L} * \int_0^L \left(\dot{\alpha} + \frac{(\dot{x} - \dot{\alpha})y}{L} \right)^2 dy$$

$$spring\ KE = \frac{1}{2}\left(\frac{\mu}{3}\dot{\alpha}^2 + \frac{\mu}{3}\dot{x}\dot{\alpha} + \frac{\mu}{3}\dot{x}^2\right)$$

- μ/L is the linear density of the spring (constant), and the mass of a small element corresponding to dy is μ/L*dy
- The term in the parenthesis of the integral is the speed of the element dy
- In that case you can find that if the spring is not displaced ($\dot{\alpha} = 0$) then we have the classical description with 1/3 of its mass at the top:

$$spring\ KE = \frac{1}{2}\left(\frac{\mu}{3}\dot{x}^2\right)$$

- In all other cases, the actual mass is fully represented. In fact there are three (equal in that case) components of the spring mass, one being linked to both tip and butt.

Lagrangian equations

- Starting with the expression of the Lagrangian equation, partial derivatives of it can be used to create the expression, which equals the forces applied at the point corresponding to the variables: the butt of the spring (α) and the rod-tip (x). Since there is no external force applied to the rod-tip, the equation is conservative (0). At the butt end of the spring (α), force is generated by the caster:

$$L = W_{kinetic} - W_{elastic}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\alpha}}\right) - \left(\frac{\partial L}{\partial \alpha}\right) = \sum F_{\alpha}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \left(\frac{\partial L}{\partial x}\right) = \sum F_{x} = 0$$

Equations coming out from the calculation process

- Equation derived from the variable x:

$$\left(M + \frac{\mu}{3}\right)\ddot{x} + \frac{\mu}{6}\ddot{\alpha} + k(x - \alpha) = 0$$

- Equation derived from the variable α:

$$\left(m + \frac{\mu}{3}\right)\ddot{\alpha} + \frac{\mu}{6}\ddot{x} - k(x - \alpha) = F_{\alpha}$$

- Sum of these two equations (one of Newton’s laws):

$$m\ddot{\alpha} + M\ddot{x} + \mu\frac{(\ddot{\alpha} + \ddot{x})}{2} = F_{\alpha}$$

Conservative case (free deceleration)

- Setting $F\alpha = 0$ allows the relationship between tip and butt acceleration to be calculated. Then one can deduce the acceleration of the marble with the first Lagrangian equation by eliminating the butt acceleration from it:

$$\ddot{\alpha} = -\frac{M + \mu/2}{m + \mu/2}\,\ddot{x} \qquad \ddot{x} = \frac{k(\alpha - x)}{\frac{\mu}{3} + M\frac{m + \mu/3}{m + \mu/2}}$$

- Since the marble keeps on accelerating because α>x as the spring is compressed, the butt decelerates by necessity (negative sign in the first equation). The rate of this deceleration increases with the weight of the marble and decreases with the weight of the reel mass.
- In terms of modeling, m is pretty small and M is similar to μ, which gives the following order of magnitude; showing a fairly rapid deceleration rate:

$$\ddot{\alpha} \simeq -3\,\ddot{x}$$

Adaptation of some equations to fly rods

- Scaling masses for the basic model
- Model adaptation to real rods, and shortcuts
- Angular parameters
- Self deceleration amplitude for a fly rod

Scaling masses for the spring & marble model

- Starting with a practical case. A rod for a number 6 line has a stiffness in the range of 1 N/m, so if we want to cast 10 grams, its equivalent mass would roughly have to correspond to 3.3 grams to have the fundamental frequency of a rod around a little bit less than 3 Hz (2.77 Hz in this case).

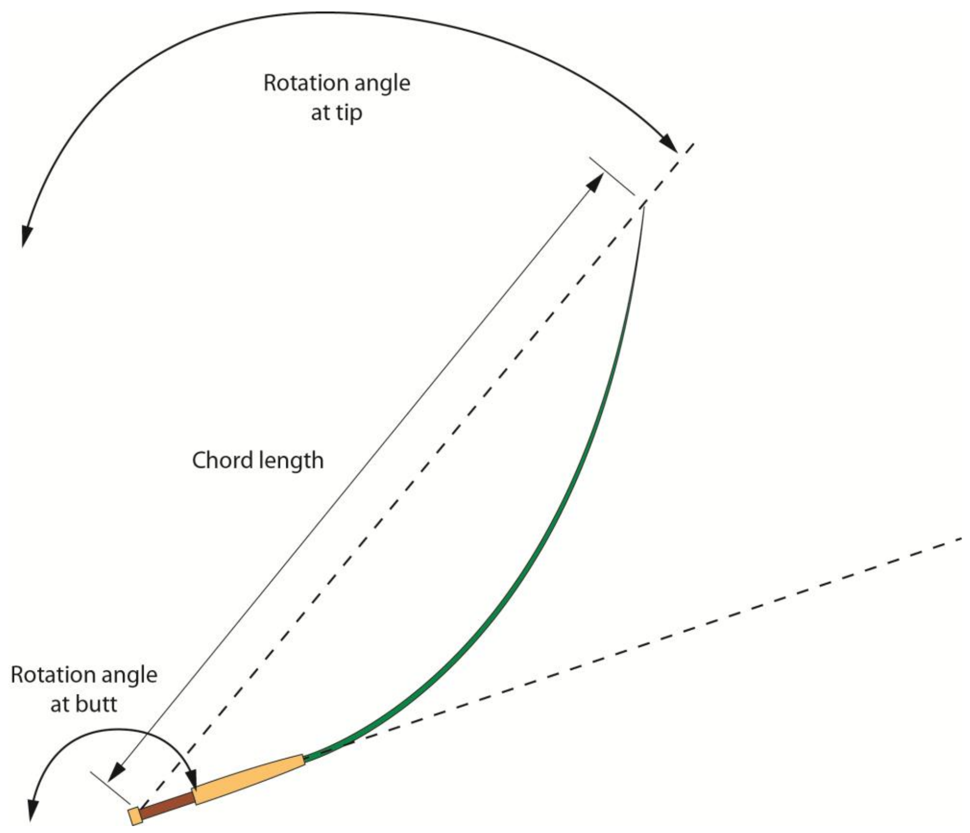
$$frequency = \frac{1}{2\pi}\sqrt{\frac{k}{\frac{\mu}{3}}}$$

- Referring back to the spring means that its mass (μ) is about 10 grams also (3*3.3).

- The reel has a relatively large mass but a small inertia, and we have to scale its mass so that it is representative of the inertia encountered with an actual fly-rod and fly-line (10 grams). Even if we assume that rotation is at the elbow joint, the inertia from the reel is moderate in comparison with either the inertia of the line or the inertia of the rod, which are both in the 100 gm^2 range and more for a number 6 rod (example), whilst the reel weighs about 15 gm^2 for this type of rod.

Model adaptation to real rods and shortcuts (1/3)

- There is one more dimension to consider so we have to introduce another variable like the chord length, polar coordinates, or angles instead of displacements.
- The difficulty lies in the relationship between the components of the Lagrangian equation and the chord length. In fact all parameters like stiffness and equivalent masses depend on the chord length but there is no simple relationship than can be processed through the methodology that is used.



Model adaptation to real rods and shortcuts (2/3)

- So, in practice, one possibility is to map parameters (stiffness, measured as an angular characteristic), and equivalent masses, and use this mapping in the numerical solution program. A more correct solution would be to use sophisticated software (professional), which can handle the required level of complexity.
- The advantage of such a shortcut (mapping) is that equations are maintained as they are but adapted for angular variables, and the new equations are interpreted (same as for the spring & marble model).
- It is possible, with an adequate methodology, to estimate the equivalent masses (inertia) as the rod deflects, including the global variation of inertia (swing weight of the rod).
- Surprisingly, these equivalent inertia represent approximately 1/3 each of the total inertia of the rod, and this explains why a spring can give a good estimate of the physics of the fly rod cast.
- Optimization can be achieved through rod design to maximize or minimize each of the equivalent inertia, depending on the objectives in terms of rod behaviour.

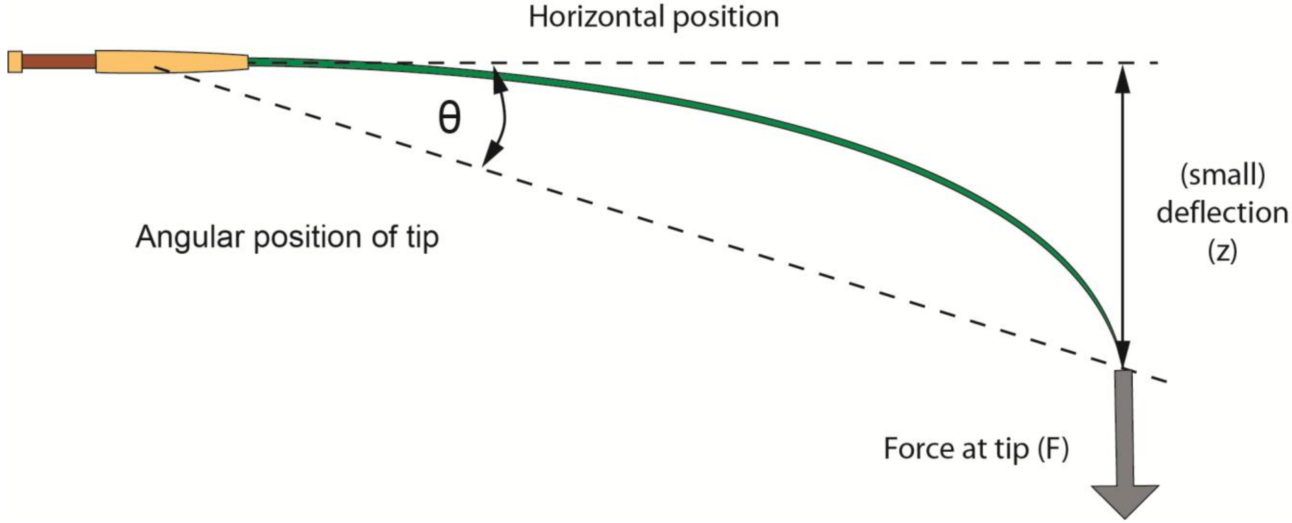
Model adaptation to real rods and shortcuts (3/3)

- A rod is a non-linear spring and its angular stiffness for any load can be estimated as follows:

$$K_w = K_{wlin}\,(1 + nl\,\theta_L^2)$$

- There is a linear component (Nm/rad) and then a non-linear factor (*nl*), which depends on the amount of deflection of the rod and the direction of the force applied to it. θ_L is the rotation angle of the rod-tip. This results in a "Duffing Equation" for the rod-tip motion.
- For a complete description of the rod, you need to take the chord of the deflected rod under consideration in the equations.

Angular parameters



- With the fly-rod in a horizontal position and a force (F) applied at the tip. Deflection is "z" and stiffness can be estimated from the ratio F/z (N/m) = k
- To consider the angular stiffness, assume that there is a small deflection and the moment of the force F is F*L: L being the length of the rod. Then its angular stiffness is FL/θ = Kw: θ being the angular position of the tip as illustrated above. The dimension of this scalar quantity is Nm/rad
- In the linear domain represented by small deflections, we can write that Kw ≃ k*L2, because z ≃ θL
- In practice, we use this to estimate rod stiffness with the F/z relationship

Self deceleration equation for a rod (1/3)

- The following can be used as a guideline. Instead of mass, we speak of inertia and the notation used is:
  - J_{otip} = equivalent inertia at tip end
  - J_{obutt} = equivalent inertia at butt end
  - $J_{otransfer}$ = equivalent transfer inertia
- Hence SW (swing weight) = $J_{otip} + J_{otransfer} + J_{obutt}$
- If K_w is the angular stiffness, then by comparison to the spring $K_{wlin} = k\,L^2$ (square of rod length).
- The fundamental frequency of the rod is: $$frequency = \frac{1}{2\pi}\sqrt{\frac{K_{wlin}}{J_{otip}}}$$
- The inertia of the reel is noted J_{reel}

Self deceleration equation for a rod (2/3)

- Below is the equation for the deceleration of the butt for a limited deflection (for a large deflection, L is replaced by the chord) applied to a fly-rod. This is essentially a guide since it has been simplified by making some assumptions:

$$\ddot{\alpha} = -\frac{ML^2 + J_{otip} + \frac{J_{otransfer}}{2}}{J_{reel} + J_{obutt} + \frac{J_{otransfer}}{2}}\ddot{x}$$

- The influence of the line (***M***) is large and it dictates the deceleration of the butt.
- The role of the various equivalent inertias can be seen. A tip action rod (small J_{otip}) or a butt action rod (high J_{otip}) have an influence on the SDM. If a stick (rod of constant thickness) is used to cast an apple, then the deceleration of the butt is maximized since J_{otip} represents about 77% of the SW and J_{obutt} something like only 4%. The remaining 19% SW corresponds to the equivalent transfer mass; fairly small in comparison to a fly rod.
- The larger inertia of the reel is, the lower the deceleration rate tends to be. Leave it in your pocket if you can!

Self deceleration equation for a rod (3/3)

- In fact a fly rod is directed by another harmonic under these circumstances (called free-free ends); with two nodes, one in the tip and the other in the butt. The one in the butt is influenced by the mass of the reel, so if the reel is heavy, the node can be lower than the hand of the caster and this gives an uncomfortable feel. The best way to control the behaviour of the rod is to have the node in the butt just under the grip.
- Therefore, it is recommended the mass of the reel is limited to something like one, to one and a half times, the mass of the rod.