Class #12 Wednesday 23 February 2011

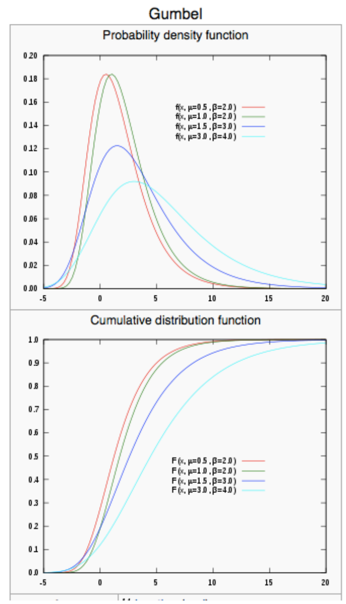
1

What did we cover last time?

- Continuous distributions
  - Beta Distribution
  - Extreme Value Distributions (Extreme Types Theorem), Gumbel, Weibull, & Freshet
  - Mixed Distribution
- Measures of Fit
  - Superposition
  - Quantile-Quantile Plots
- Parameter Fit
  - Maximum Likelihood
- Open source software for statistics - Octave & R have components.
- TEST, Get Thursday 3Mar, return 8Mar Tues.
- Homework 3, hand out. From text book

Gumbel

2



$$f(x) = \frac{e^{-\frac{x-\zeta}{\beta}} \bullet e^{-e^{-\frac{x-\zeta}{\beta}}}}{\beta}$$

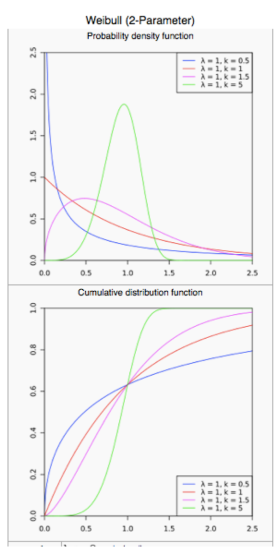
Weibull

3

$$f(T) = \frac{\beta}{\eta}\left(\frac{T-\gamma}{\eta}\right)^{\beta-1} e^{-\left(\frac{T-\gamma}{\eta}\right)^{\beta}}$$

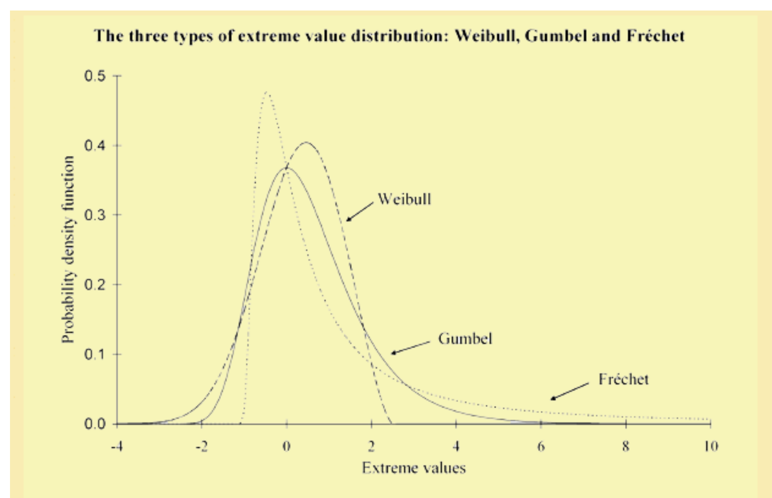
and,

- η = scale parameter,
- β = shape parameter (or slope),
- γ = location parameter.



Compare the three extreme value distributions

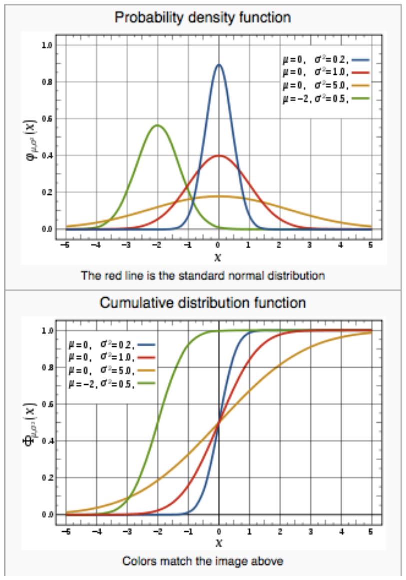
4

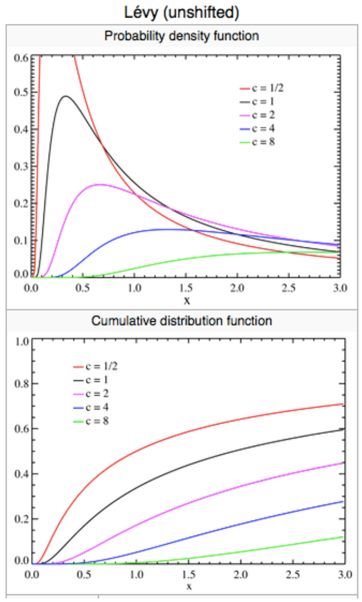


Longin F. (1996) "The asymptotic distribution of extreme stock market returns" *Journal of Business*, N°63, pp 383-408.

http://www.longin.fr/Recherche_Publications/recherche_publications_asymptotic_distribution.htm

Gaussian vs Levy





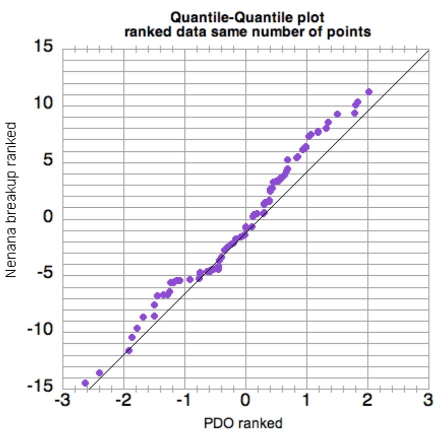
Quantile-quantile plots

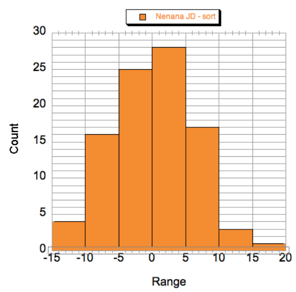
⁕ It is is a <u>graphical method</u> for comparing two <u>probability distributions</u> by plotting their quantiles against each other. Point (x,y), plot x quantile versus y quantile.

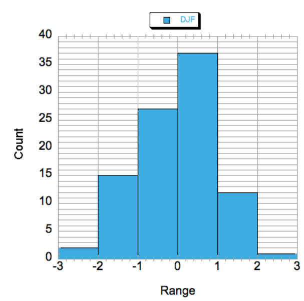
⁕ Recipe

- ⁕ Rank data, decide how many pieces to divide into, determine quantiles, plot
- ⁕ If the data sets have the same size, the q-q plot is essentially a plot of sorted data set 1 against sorted data set 2. If the data sets are not of equal size, the quantiles are usually picked to correspond to the sorted values from the smaller data set and then the quantiles for the larger data set are interpolated.

Quantile-quantile plots







3.6 Parameter fit using Maximum Likelihood

⁕(A) Likelihood Function cont...

⁕ BOARD - how to solve...

⁕(B) Newton-Raphson Method

⁕ Gaussian MLEs were easy to calculate, usually calculate iteratively. Calculate roots.

⁕ BOARD - example and how to solve...

⁕ Example 4.12 Gamma Function

$$f(x) = \frac{(x/\beta)^{\alpha-1}\exp(-a/\beta)}{\beta\Gamma(\alpha)}, \quad x,\alpha,\beta > 0 (4.38)$$

⁕(C) Expectation-Maximization (EM) Method

⁕ Use for more than 3 parameters, more of an idea than a formulaic process. Book points to references for details on process.

3.7 Statistical Simulation

⁕ Statistical Simulation: Generate 'seemingly' random numbers based on given PDF, EX: SAT forcing for an ocean model.
⁕ Random number generator: really pseudo-random

⁕(A) Uniform Random number generation
⁕ Generate random uncorrelated samples between 0 and 1.
⁕ How does this work? BOARD,

4.0 Hypothesis Testing

4.1 Background
⁕ Parametric versus non-parametric testing
⁕ Parametric - associated with a particular distribution so you make inferences about the distribution
⁕ Non-parametric - a) construct a test so that the distribution of the data is not important (classical approach) or b) important aspects of the data are inferred directly from the data (resampling approach).
⁕ Sampling Distribution - is the distribution of a given statistic based on a random sample of size n. The sampling distribution depends on the underlying distribution of the population, the statistic being considered, and the sample size used.

4.1 Background continued..

⁕ The sampling distribution is the probability distribution describing the batch-to-batch variations of the statistic.
⁕ 5 Elements of Any Hypothesis Test.
1. Identify a test statistic useful for question and data you are using. The test statistic will be the subject of the test. Parametric - will be a parameter, Nonparametric - more flexible.
2.Define a *null hypothesis*, usually H_O, a proposed idea that you hope to reject
3. Define an *alternative hypothesis*, H_A, such as 'H_O is not true'
4. Obtain the *null distribution*, sampling distribution for the test statistic
5. Compared observed test statistic to null dist.

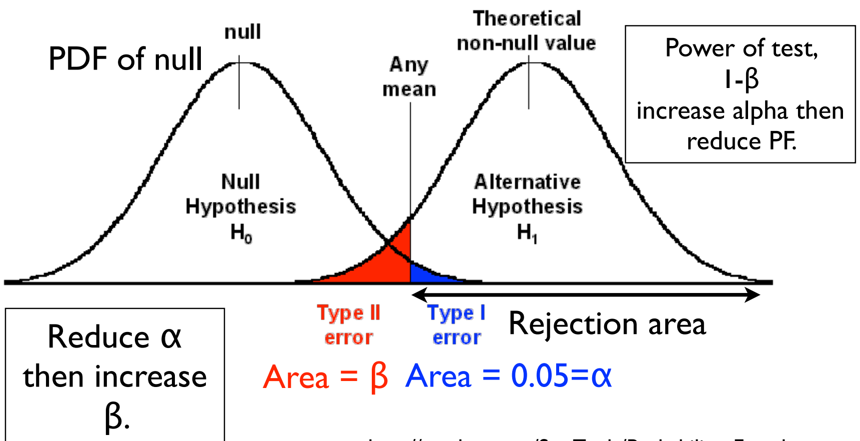
4.0 Hypothesis Testing

4.1 Background continued..
⁕ Test Levels and p values
⁕ 'The sufficiently improbable region' of the null distribution, aka rejection level, or level of test. Define test level before hand (5% typical for Meteorology). The null hypothesis is rejected if the p value is less than or equal the test value.
⁕ Error types and the power of a Test
⁕ **Type I Error**: Falsely reject null hypothesis when it is true. It's probability is α.
⁕ **Type II Error**: Null hypothesis is not rejected but it is false. It's probability is β.

4.1 Background continued..

❋ Error types continued...

❋ Test conducted at 5% level. A test statistic falling to the right of critical value => reject null hypothesis



http://amchang.net/StatTools/Probability_Exp.php

4.1 Background continued..

❋ One-sided vs Two-sided tests (tailed)

❋ One-sided (like previous figure) looks at one side for some physical reason.

❋ Two-sided apply when either very large or very small values of the test statistic are unfavorable to null hypothesis. Apply to such a case as 'H_0 is not true'. Sum of the two probabilities add up to alpha, so if α=0.05 then 0.025 on right and left are the rejection regions.

❋ Confidence intervals - tells you the reliability of an estimate. It is the inverse of the test statistic because it gives you values of the test statistic that would not fall into the rejection region.

EXAMPLE 5.1 Hypothesis test example

4.2 Parametric Tests

❋(A) One-sample t-test, very popular since based on the Gaussian. Student's t-test, based on the t distribution:

$$t = \frac{\bar{x} - \mu_o}{[\hat{Var}(\bar{x})]^{1/2}} \quad (5.3)$$

$$\hat{Var}(\bar{x}) = s^2 / n \quad (5.4)$$

❋ t distribution like Gaussian but with heavier tails, & is controlled by parameter v (degrees of freedom). v=n-1 (n= independent observations)

❋ Can read t values off of tables, use equations (available from Abramowitz and Stegun) or approximate using a Gaussian distribution if n big.

4.2 Parametric Tests continued...

❋(B) Tests for Differences of Mean under Independence

❋ Compare two samples means, are they different. Example: GCM experiment no arctic sea ice and Control simulation with late 20th century sea ice. *Null Hypothesis-Difference is zero between means.* Yields test statistic:

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - E[\bar{x}_1 - \bar{x}_2]}{[\hat{Var}(\bar{x}_1 - \bar{x}_2)]^{1/2}} \quad (5.5)$$

4.2 Parametric Tests continued...

❋(C) Tests for Differences of Mean for Paired Samples

❋ Blah Blah Blah

4.2 Parametric Tests continued...

❋(D) Tests for Differences of Mean under serial correlation

❋ Reduce the number of degrees of freedom