ATM 645 Math Review
Fall 2014
Given out: Tuesday Sep 9, 2014
Due: Friday Sept 19, 2014

1. Let $\vec{A} = \nabla\phi = 8x\hat{i} + 3y^2\hat{j}$. If you know that $\phi(1,1) = 8$ and $\phi(0,1) = 4$, derive a functional expression for $\phi(x,y)$.

2. Prove the vector identities in (a)-(c) letting $\vec{V} = u\hat{i} + v\hat{j} + w\hat{k}$ and $\nabla = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$:

a) $\nabla \bullet (\nabla \times \vec{V}) = 0$

b) $(\vec{V} \bullet \nabla)\vec{V} = (1/2)\nabla(\vec{V} \bullet \vec{V}) - \vec{V} \times (\nabla \times \vec{V})$

c) $\nabla \bullet (f\vec{V}) = f(\nabla \bullet \vec{V}) + \vec{V} \bullet \nabla f$

3. Demonstrate that $\vec{A} \bullet (\vec{B} \times \vec{C}) = -\vec{B} \bullet (\vec{A} \times \vec{C})$

4. Using the horizontal wind vector

$$\vec{u}(x,y,z,t) = \left[C \sin\left(\frac{2\pi}{L} x \right) + \frac{C}{4000} y \right] \hat{i}$$

where C=10m s^{-1} and L=1 x 10^6 m

a) Plot the $\hat{i}$ component of $\vec{u}$ for $-1000\,km \le x \le 1000\,km$ at y=-500, 0, and 500 km.

b) Calculate $\frac{\partial \vec{u}}{\partial x}$

c) Plot $\frac{\partial \vec{u}}{\partial x}$ for $-1000\,km \le x \le 1000\,km$

d) Describe the relationship between $\vec{u}$ and $\frac{\partial \vec{u}}{\partial x}$ shown in the plots from parts a) and c). Is this consistent with your understanding of the physical meaning of $\frac{\partial \vec{u}}{\partial x}$?