Homework 6 ATM 645 Fall 2014
Assigned: Wed 14 Oct, 2014, Due: Friday 24 Oct 2014.

1. (Holton 4.1) What is the circulation about a square of 1000 m on a side for an easterly (i.e. westward flowing) wind that decreases in magnitude toward the north at a rate of 10 m/s per 500 km? What is the mean relative vorticity in the square?

2. (Holton 4.2) A cylindrical column of air at 30˚N with radius 100 km expands to twice its original radius. If the air is initially at rest, what is the mean tangential velocity at the perimeter after expansion?

3. (Holton 4.4) An air column at 60˚N with initial vorticity =0 initially stretches from the surface to a fixed tropopause at 10km height. If the air column moves until it is over a mountain barrier 2.5 km high at 45ΔN, what is its absolute vorticity and relative vorticity as it passes the mountain top assuming that the flow satisfies the barotropic potential vorticity equation?

4. (Holton 4.7) Compute the rate of change of circulation about a square in the (x,y) plane with corners at (0,0), (0,L), (L,L), and (L,0) if temperature increases eastward at a rate of 1˚C over 200 km and pressure increases northward at a rate if 1 hPa over 200 km. Let L=1000 km and the pressure at the point (0,0) be 1000 hPa.

5. (Holton 4.11) A cyclonic vortex is in cyclostropic balance with a tangential velocity profile given by the expression $V = V_0 (r / r_0)^n$ where V_0 is the tangential velocity component at distance r_0 from the vortex center. Compute the circulation about a streamline at radius r, the vorticity at radius r, and the pressure at radius r. (Let P_0 be the pressure at r_0 and assume the density is a constant).

6. (Holton 4.12) A westerly zonal flow at 45˚N is forced to rise adiabatically over a north-south oriented mountain barrier. Before striking the mountain, the westerly wind increases linearly toward the south at a rate of 10 m/s per 1000 km. The crest of the mountain range is at 800 hPa and the tropopause, located at 300 hPa, remains undisturbed. What is the initial relative vorticity of the air? What is its relative vorticity when it reaches the crest if it is deflected 5˚ latitude toward the south during the forced ascent? If the current assumes a uniform speed of 20 m/s during its ascent to the crest, what is the radius of curvature of the streamlines at the crest?

7. (Holton 4.14) (a) How far must a zonal ring of air initially at rest with respect to the earth's surface at 60˚latitude and 100-km height be displaced latitudinally in order to acquire an easterly (east to west) component of 10 m/s with respect to the earth's surface? (b) To what height must it be displaced vertically in order to acquire the same velocity? Assume a frictionless atmosphere.

8. On a particular occasion the wind was 14.14 m/s from the NW near the surface and 14.14 m/s from the SW at a height of 4 km. What were the horizontal components of relative vorticity?

9. On a particular occasion the wind in the vicinity of a point 'X' was observed to be blowing from SE to NW in straight parallel lines with wind speed increasing towards the NE by 20m/s for each 500 km. What was the relative vorticity? If this was at 45˚N, what was the absolute vorticity?

10. Determine the circulation around the closed path $x^2 + y^2 = 1$ when $V = -6y\mathbf{i} + 8x\mathbf{j}$.

11. Derive the horizontal divergence and vertical component of the vorticity of the geostrophic wind assuming a) f and ρ are constants and b) $f=2\Omega\sin(y/a)$ & ρ is constant, where a = radius of the earth.

12. Show that the vorticity field for any flow satisfies: $\nabla \bullet \vec{\omega} = 0$