Dynamic Modeling of Transport Barrier Formation and Evolution

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Minimal models with radial profile of growth rates display:

1) Critical transitions with bursting behavior and multiple time scales in the dynamics
2) Similar transition dynamics from a wide variety of trigger mechanisms
3) Importance of deposition profiles
4) Reynolds stress driven flows triggering transition

==> Dynamics of transition suggests control strategies

AT meeting at GA March 11 1999

Outline

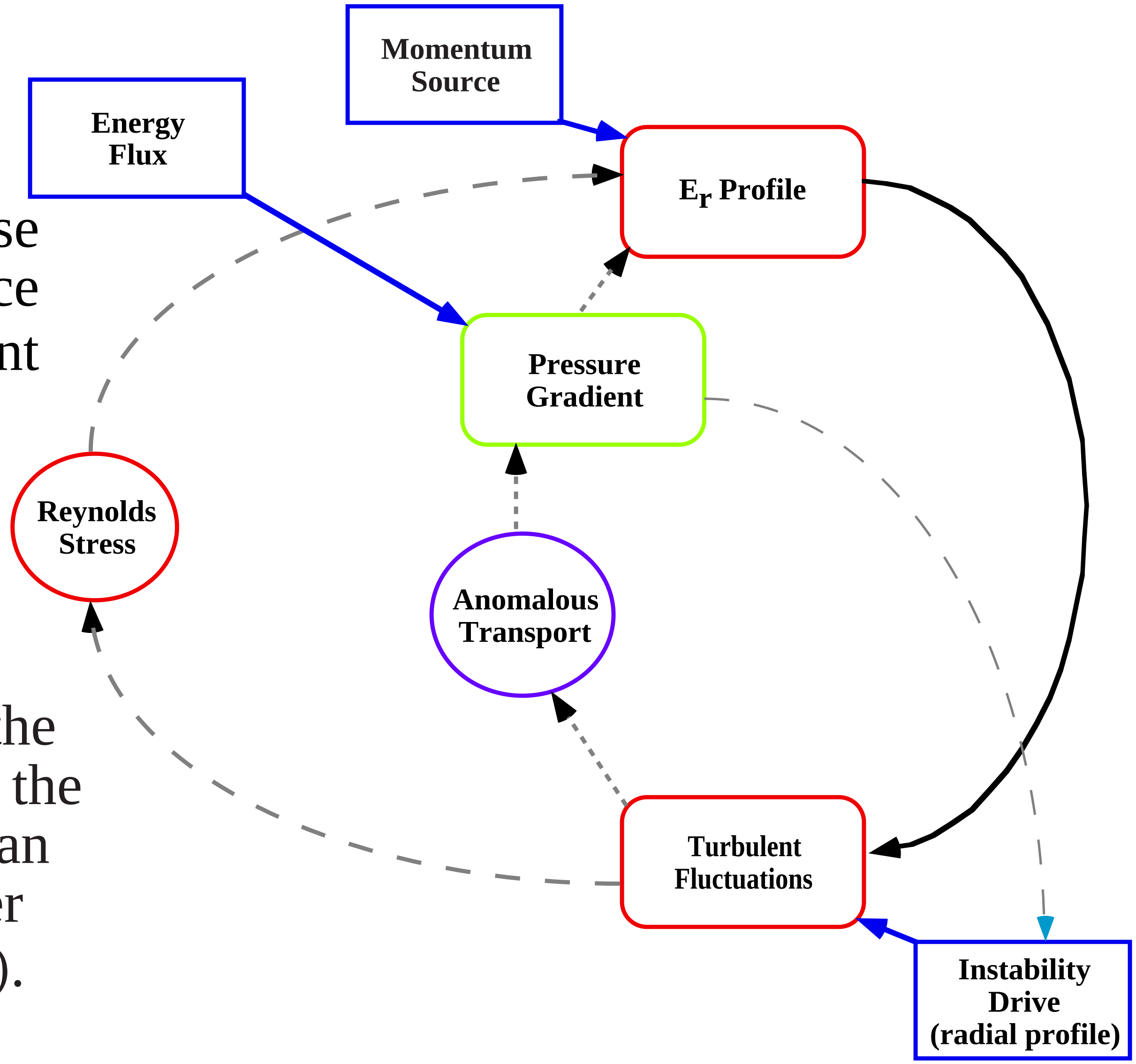
- **Motivation**
- **Basic model**
  - **Fluctuation and Pressure gradient evolution**
- **Extended transport/transition model**
  - **Basic model**
  - **Transition dynamics**
  - **Deposition profile effects**
  - **Reynolds stress driven poloidal flow**
  - **Methods for initiation**
  - **Methods for control**
- **Summary**

Motivation

- **Transitions from a reversed/weak shear mode to a high confinement (ERS, ENCS etc) mode observed in different devices suggests underlying details of initiation may differ, dynamics of transition are the same**
- **What questions can a simple dynamical model answer?**
- **Challenge to find:**
  - **threshold**
    - **threshold quantity(ies) for control (opportunity for integrated approach)**
  - **transition dynamics**
  - **barrier width and localization**
  - **mechanism for post-transition control of gradients**
  - **role of magnetic shear in transition and sustainment**

Heuristics of Model

- Radially inhomogeneous turbulent growth rates (due to magnetic shear profile) and deposition profiles can produce sheared ExB flows.



- Sheared flows increase decorrelation of turbulence $\Rightarrow$ reducing the turbulent amplitude and transport.

- Coupling exists between the turbulent fluctuations and the pressure gradient which can also feed back on the other sheared flows (V_θ and V_ϕ).

Shear Flows Effect on Transport

4 mechanisms for reduction of turbulent transport

- **Shear modification of linear drive (plasmas +)**
- **Shear decorrelation of turbulence (all turbulent systems)**
- **Phase shift between advected and advecting quantities**
- **Shear decorrelation of transport events (avalanches)**

Linear Drive $\propto$ Frequency
Shear Flow can change the Freq. and therefore change the drive

⇨

Turbulent Amplitude $\propto$ Drive

⇨

Turbulent Transport $\propto$ Turbulent Amplitude

Basic Internal Barrier Transition Model

Coupled nonlinear envelope equations for the fluctuations level and pressure gradient

Instability drive | Nonlinear damping | Sheared E Field stabilization

$$\frac{\partial \mathrm{E}}{\partial t} = \gamma_0 N \mathrm{E} - \alpha_1 \mathrm{E}^2 - \alpha_2 \langle V_E' \rangle^2 \mathrm{E} + D_0 \frac{\partial}{\partial x}\left(\mathrm{E} \frac{\partial \mathrm{E}}{\partial x} \right)$$

$$\frac{\partial N}{\partial t} = -\alpha_4 N + Q + D_0 \frac{\partial}{\partial x}\left(\mathrm{E} \frac{\partial N}{\partial x} \right)$$

Collisional transport | Power/particle source | Turbulent transport

where $E \equiv \left| \frac{\tilde{n}_k}{n_0} \right|^2$ $\qquad V_E' = V_\theta' - \beta V_\Phi' - \alpha N^2$ $\qquad N \equiv -\frac{a}{\langle P \rangle} \frac{dP}{dr}$

More complete transport model with transition model provides reasonable thresholds and same dynamics

- **Transport model:**

$$\frac{\partial n}{\partial t} = S_{NBI} + S_{gp} + \frac{1}{r}\frac{\partial}{\partial r}\left[rD_n \frac{\partial n}{\partial r}\right]$$

$$\frac{3}{2}\frac{\partial nT_i}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left[r\left(\chi_i n \frac{\partial T_i}{\partial r} + \frac{5}{2} D_n T_i \frac{\partial n}{\partial r}\right)\right] - D_n \frac{1}{n}\frac{\partial n}{\partial r}\frac{\partial nT_i}{\partial r} + Q^i_{NBI} + Q_{ei}\left(T_e - T_i\right)$$

$$\frac{3}{2}\frac{\partial nT_e}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left[r\left(\chi_e n \frac{\partial T_e}{\partial r} + \frac{5}{2} D_n T_e \frac{\partial n}{\partial r}\right)\right] + D_n \frac{1}{n}\frac{\partial n}{\partial r}\frac{\partial nT_i}{\partial r} + Q^e_{NBI} + Q_{Ohm} + Q_{ie}\left(T_e - T_i\right)$$

- **System is closed with a Fluctuation envelope equation**

$$\frac{\partial \varepsilon}{\partial t} = \left\{ \gamma - \alpha_1 \varepsilon - \alpha_2 \left[\frac{r}{q}\frac{\partial}{\partial r}\left(\frac{q}{r}\frac{E_r}{B_\phi}\right)\right]^2 \right\} \varepsilon + \frac{1}{r}\frac{\partial}{\partial r}\left[rD_\varepsilon \frac{\partial \varepsilon}{\partial r}\right]$$

and the equation for E_r

$$E_r = -V_\theta + V_\varphi \frac{B_\theta}{B_0} + \alpha\left[\frac{\partial T}{\partial r} + \frac{T}{N}\frac{\partial N}{\partial r}\right]$$

Additional dependencies

- **Growth rate and diffusivities as functions of plasma parameters**

$$\gamma_{\eta_i} = k_\theta \rho_s \frac{c_s}{a} f\left(\hat{S}\right)\left(\frac{a}{R}\right)^{1/2}\left(\frac{a}{L_n}+\frac{a}{L_T}\right)^{1/2}\left(\frac{T_i}{T_e}\right)^{1/2}$$

$$\chi_{i\eta_i} = k_\theta \rho_s c_s a\left(\frac{a}{R}\right)^{1/2}\left(\frac{a}{L_n}+\frac{a}{L_T}\right)^{-1/2}\left(\frac{T_i}{T_e}\right)^{1/2}\varepsilon^2$$

$$\chi_{e\eta_i} = D_{\eta_i} = k_\theta \rho_s \frac{c_s}{a\nu_e}\left(\frac{a^2}{L_n R}\right)^{1/2}\left(1+\frac{L_n}{L_T}\right)^{1/2}\left(\frac{T_i}{T_e}\right)^{1/2}\chi_{i\eta_i}$$

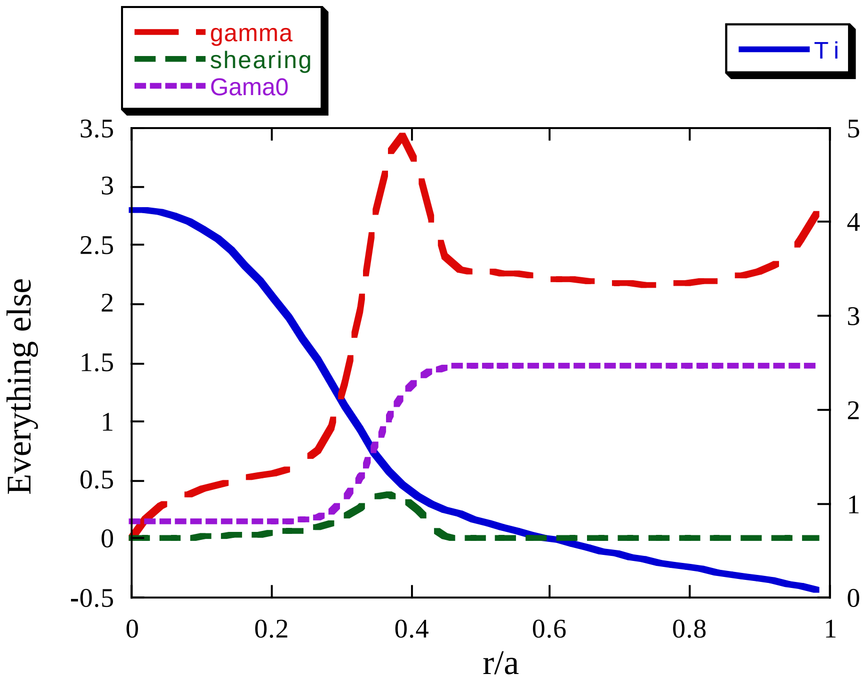
with

$$L_n = n\left(dn/dr\right)^{-1} \qquad L_T = T_i\left(dT_i/dr\right)^{-1} \qquad \alpha_2 = \left(\frac{\Delta_r}{r\Delta_\theta}\right)^2 \frac{1}{\gamma}$$

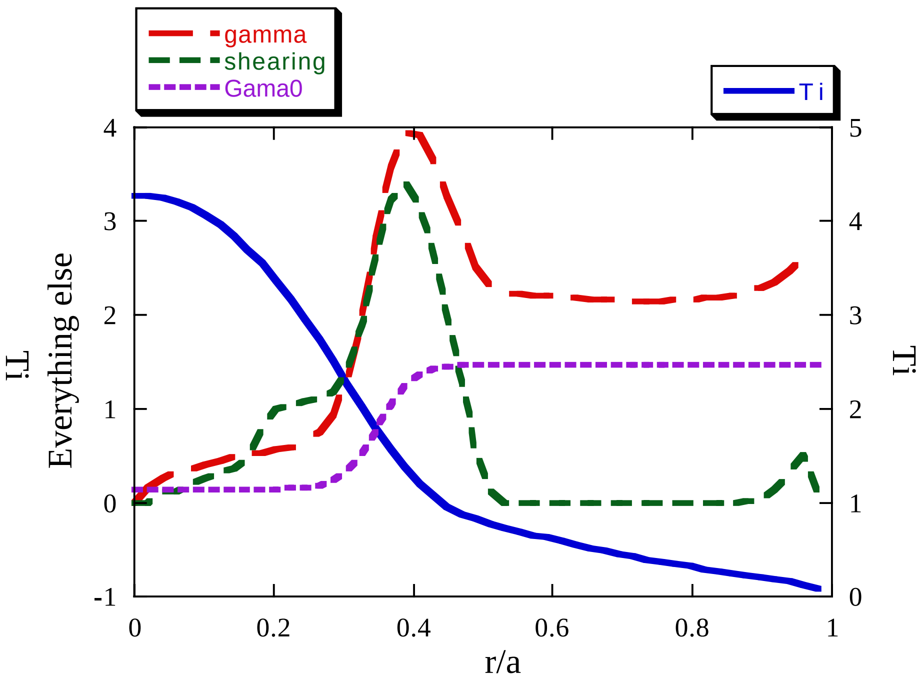
Initiation of transition occurs due to combination of q, γ and deposition profiles

- **Shear parameter:** $Shear \approx \frac{r}{q}\frac{\partial}{\partial r}\left(\frac{qE_r}{B_\varphi r}\right)$ **(see Hahm and Burrell '95)**
- **Need low growth rate, steep gradient (i.e. edge of deposition region) and appropriate q profiles**

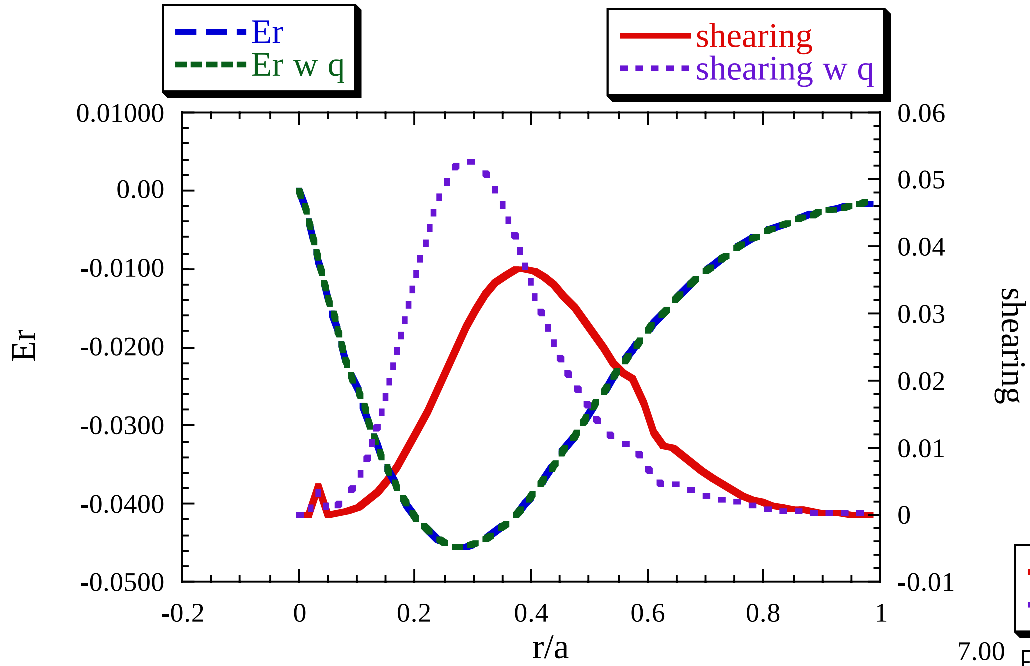
Before



After

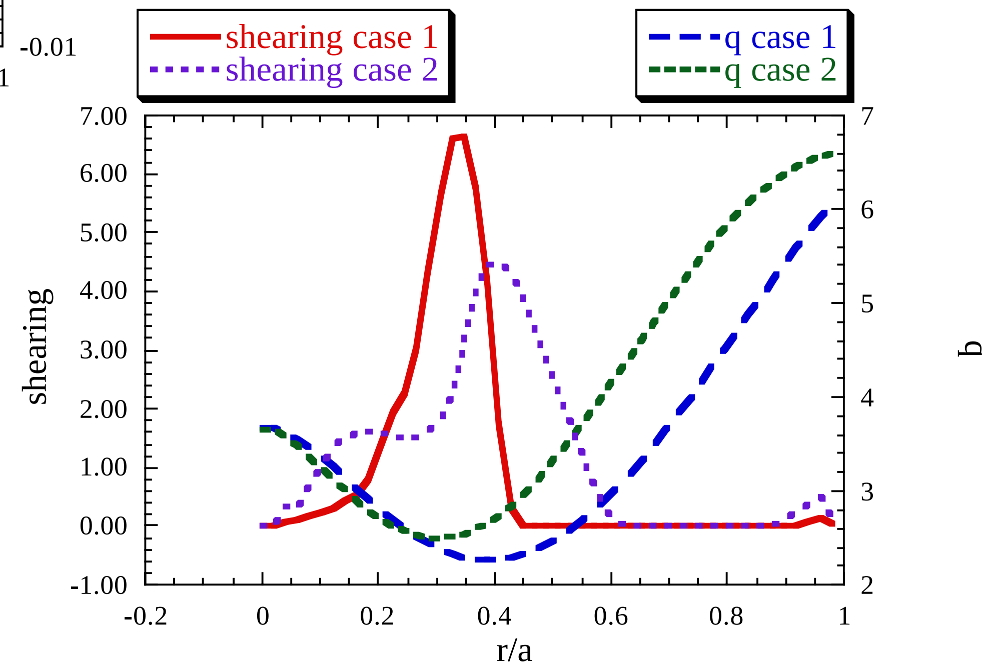


The effect of q profiles on the shearing term



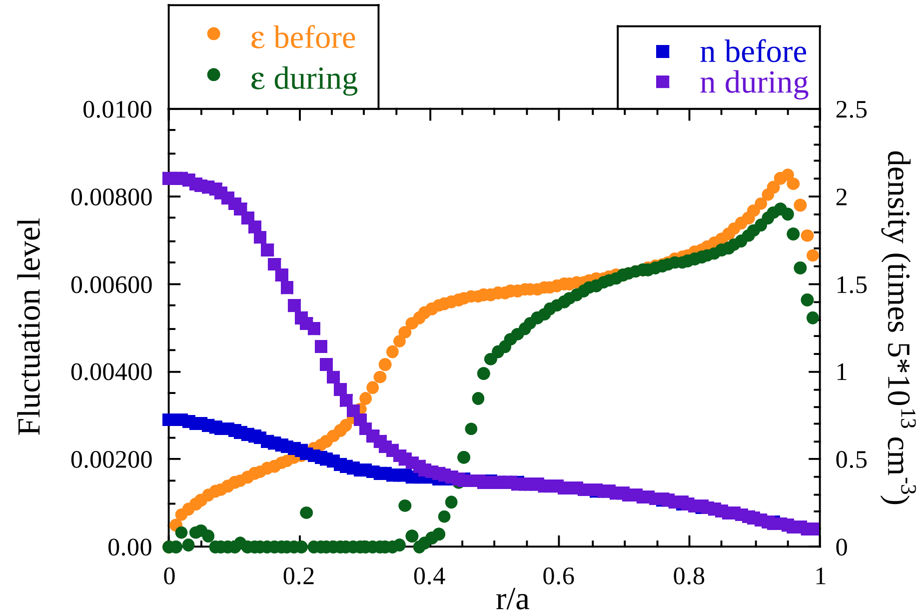
- **Shear in the q profile can both move and increase the peak in the shearing rate**

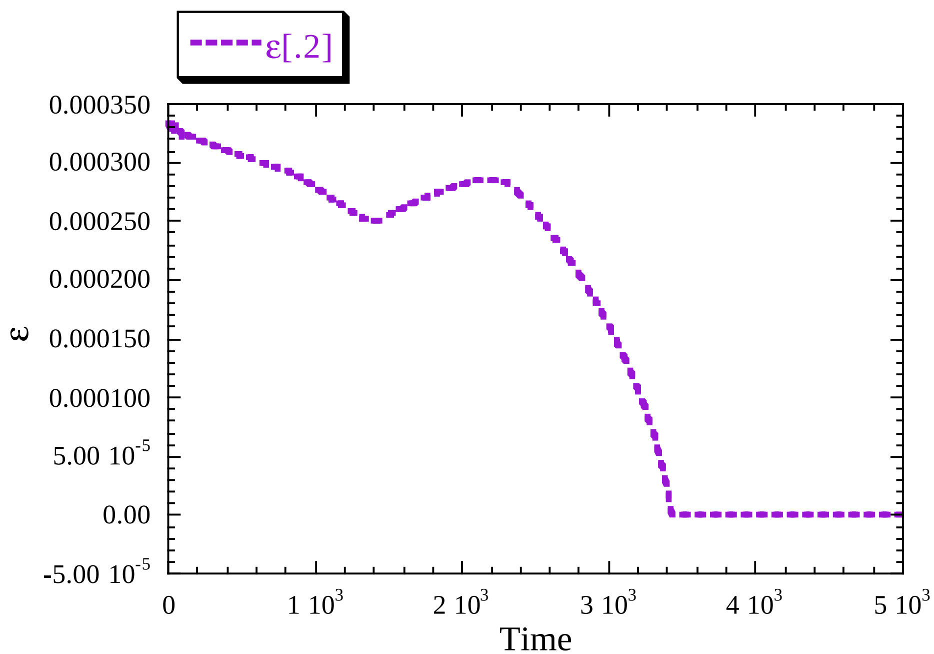
- **Change in q profile can initiate transition in different location**



Profile evolution during transition

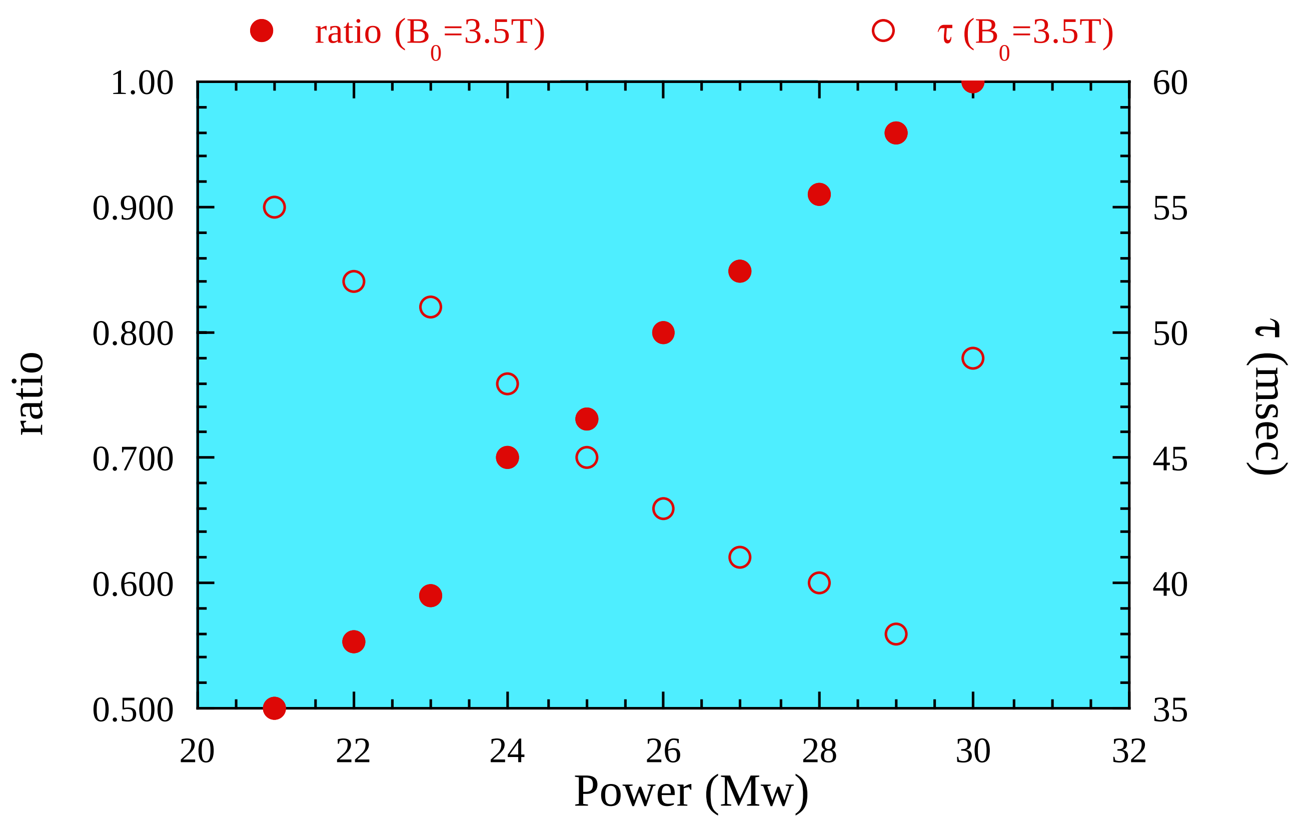
- **Fluctuations are quenched**
  - transiently reduced outside transition region due to decreased flux
- **Temperature and density profiles grow and steepen in quenched region**





Transition can be seen in confinement time at P_{th}

- **Transition occurs when ratio is 1 giving a confinement bifurcation through suppression of the fluctuation driven transport**

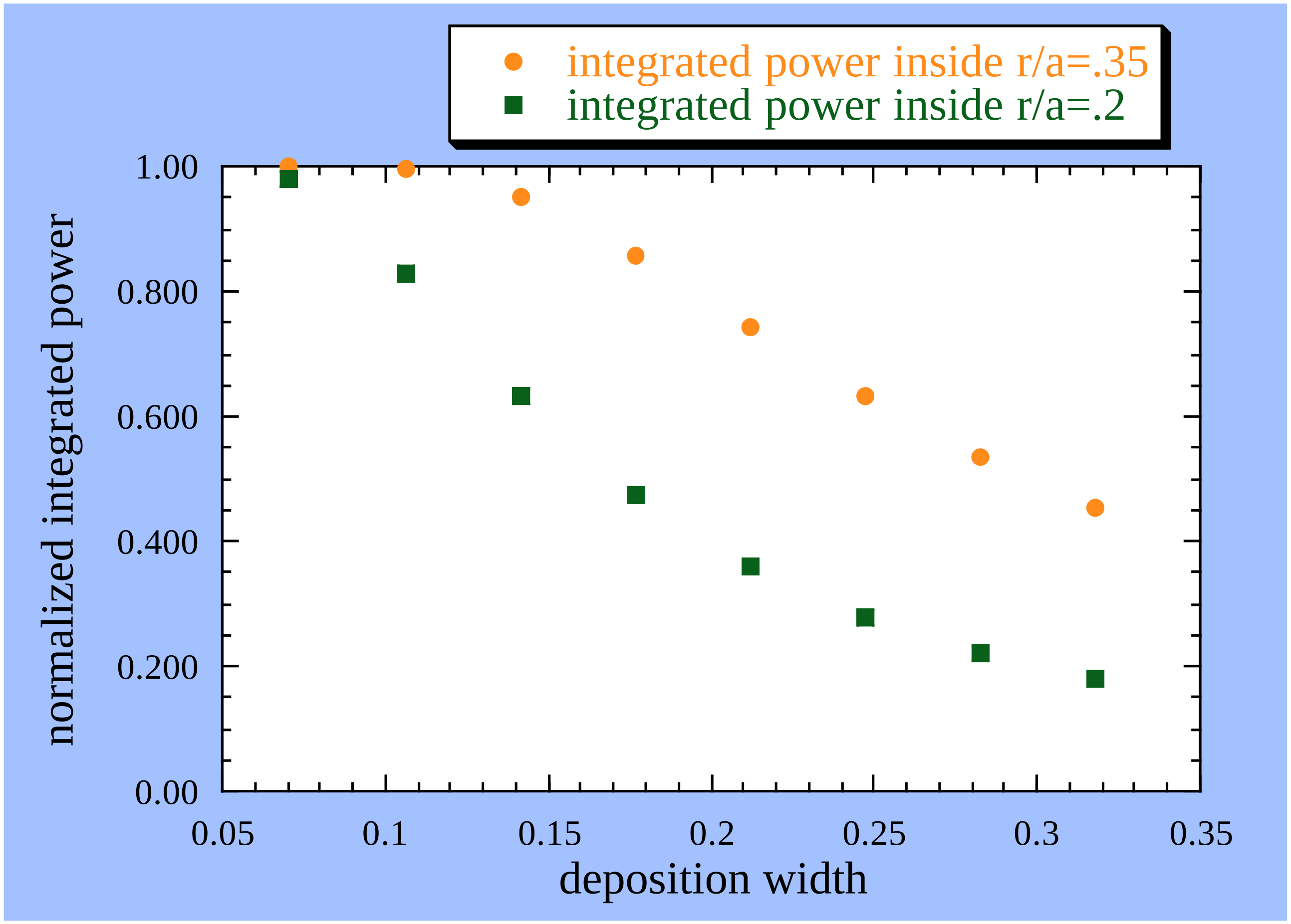


Flux through surface dependent on deposition width

- **Threshold dependent on local gradient => flux through surface by $\nabla P = \Gamma/D$ => power/particles deposited inside surface**

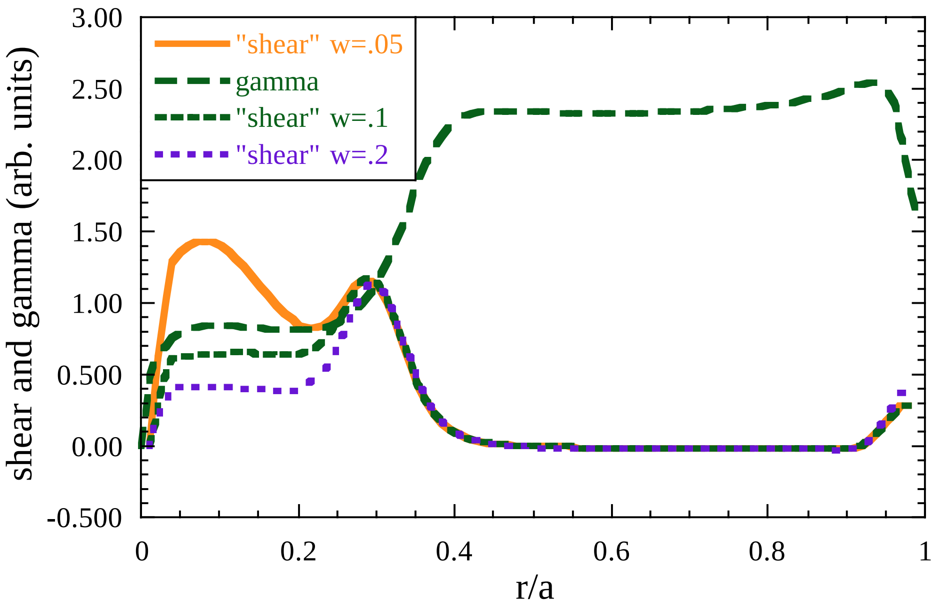
Threshold has strong dependence on deposition profile width

Possible explanation for change in TFTR power (Synakowski et al PRL 97) threshold going from D to T

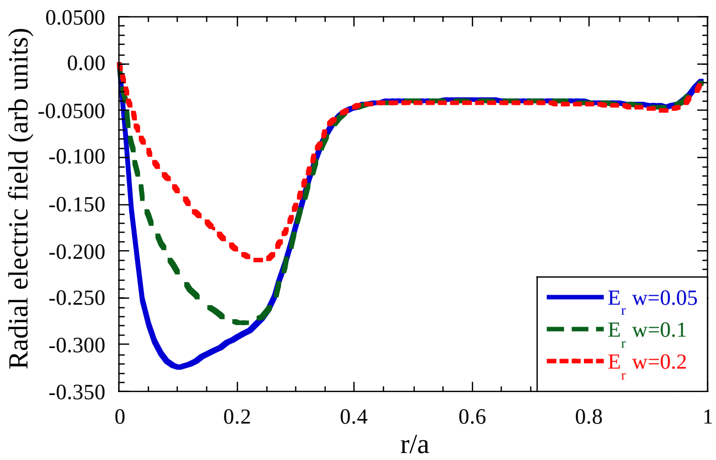


Initiation position can move with deposition profile

- **Transition can initiate well inside q_{min} for narrow deposition profiles**

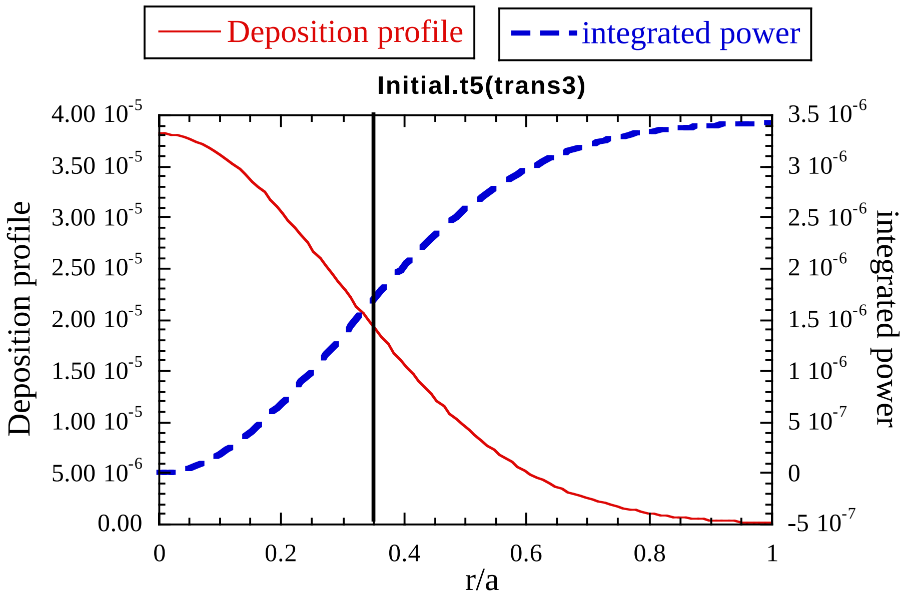


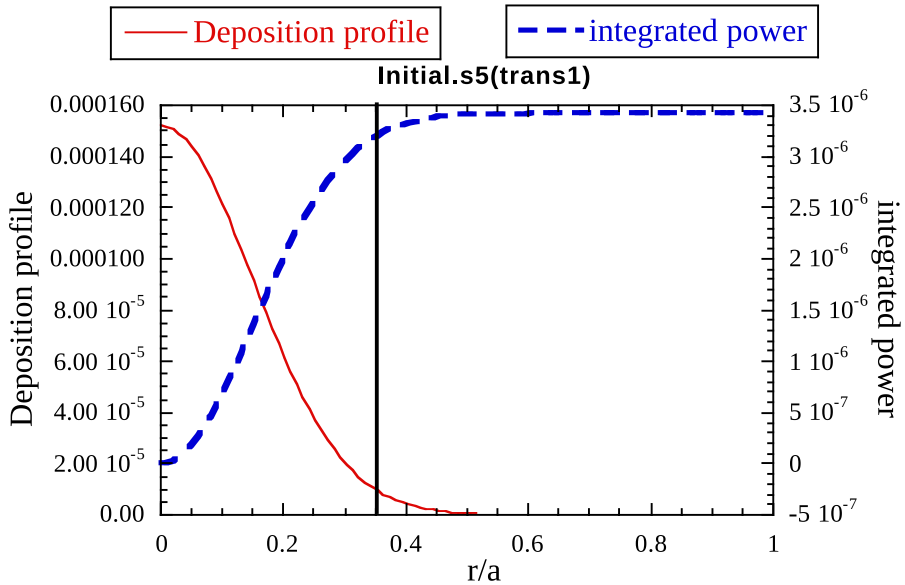
E_r profile changes



Deposition profiles important for location and extent of transitioned region

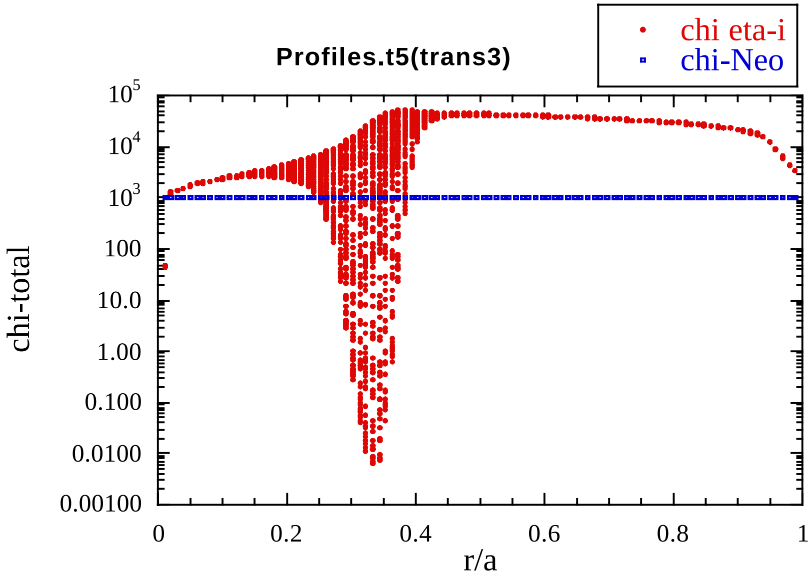
- **Same total power but different profile widths give very different flux through qmin surface**

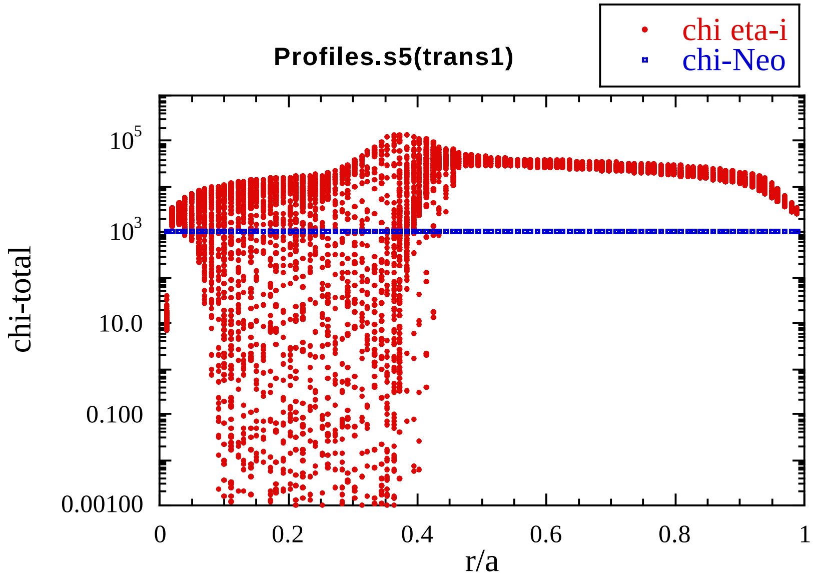




Full transition or localized transition dependent on deposition profile

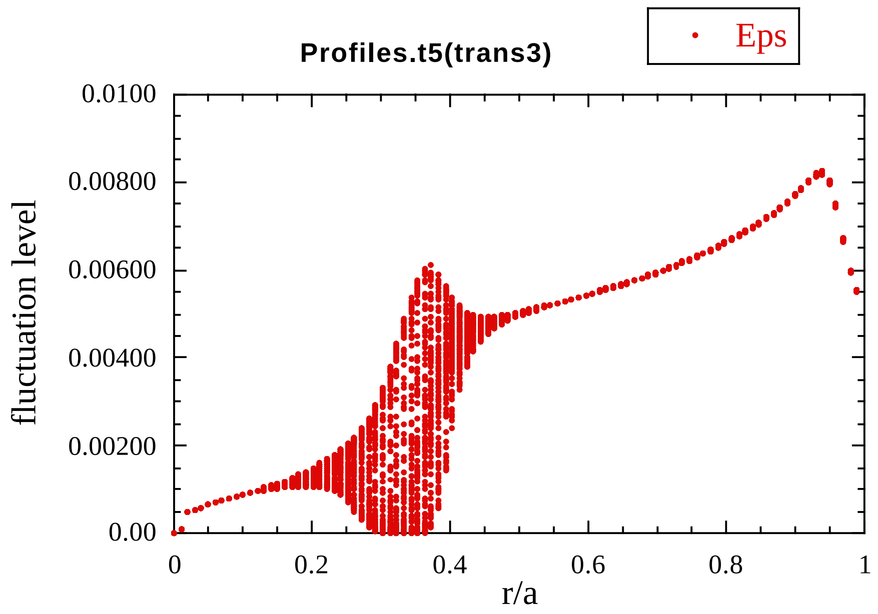
- **Broader or shifted deposition profile leads to narrow transitioned region (related to type II JT-60U ITB?)**
- **Narrower or better centered deposition profiles lead to broader ITB (related to type I JT-60U ITB?)**

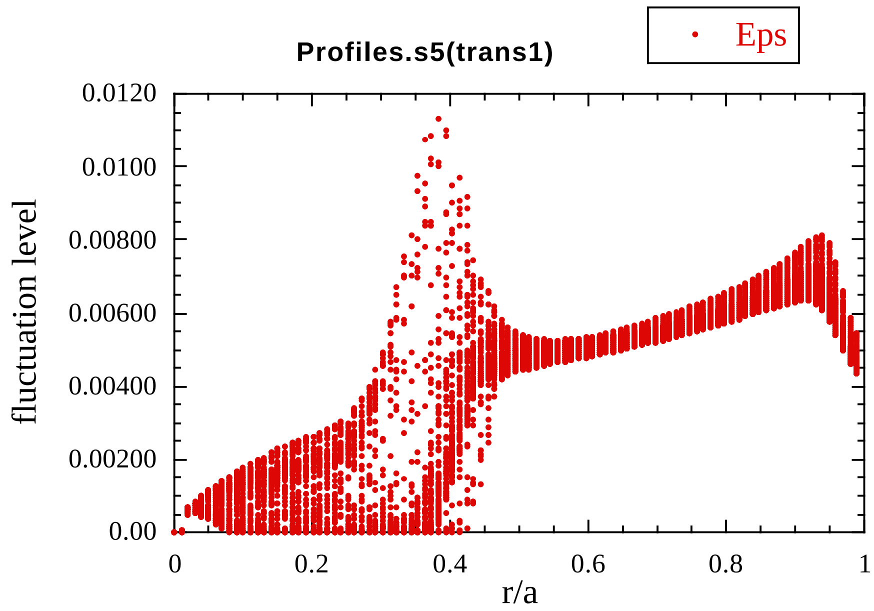




Fluctuations suppressed in over differing regions

- **Narrow region of fluctuations suppressed for broad deposition profile**
- **Broader region for narrower deposition profile**

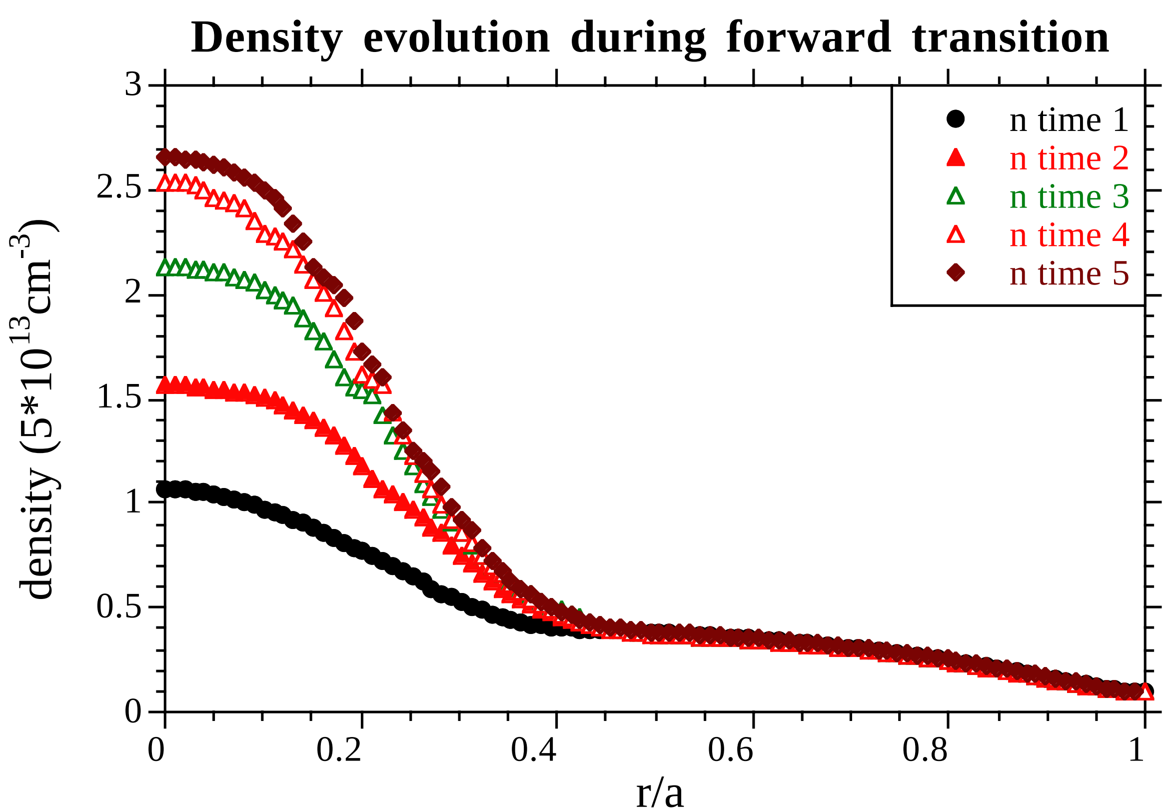




Footpoint moves out during transition

- **Steep gradient region can move passed q_{min} allowing disruptions**

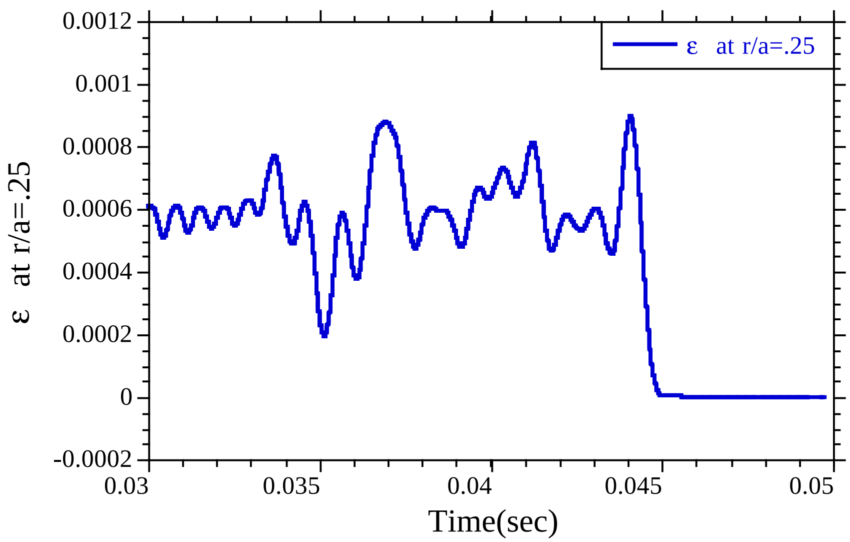
24MW transition



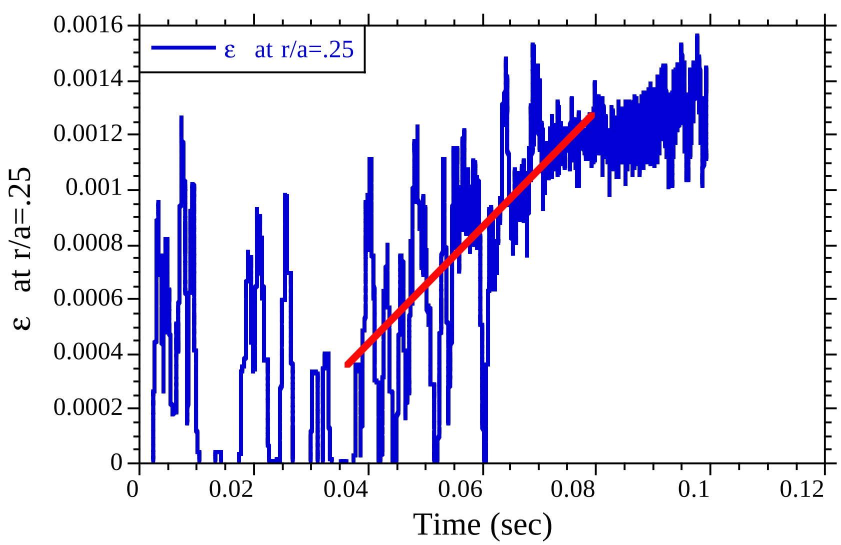
Forward transition much faster than back transition

- **Forward transition has positive feedback through gradient so bookstraps itself up**
- **Backtransition has negative feedback through increased flux due to increased diffusivities => this with growth rate effects leads to slower transitions**

Forward and back transition for 12 MW



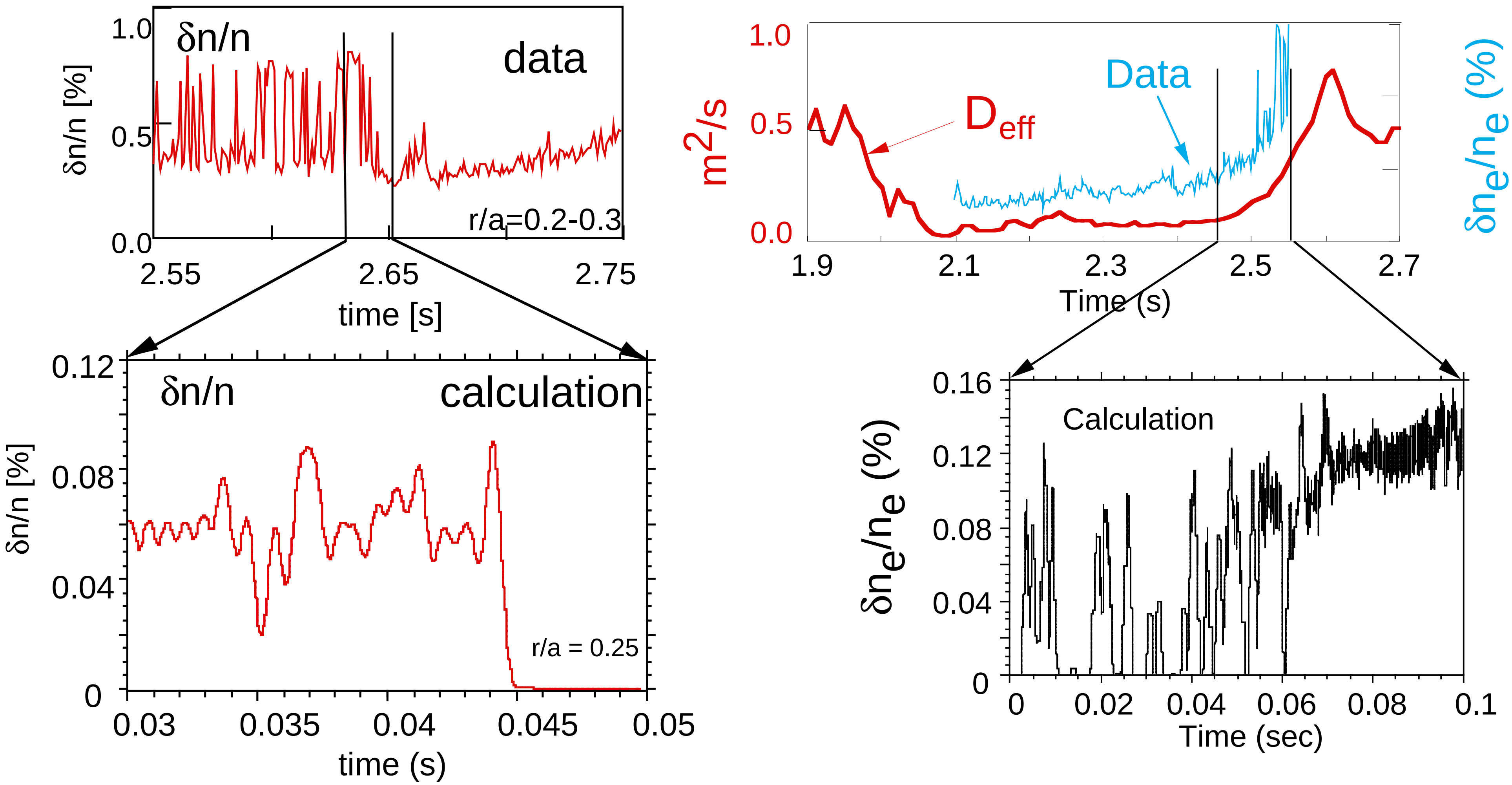
Forward transition



Backward transition

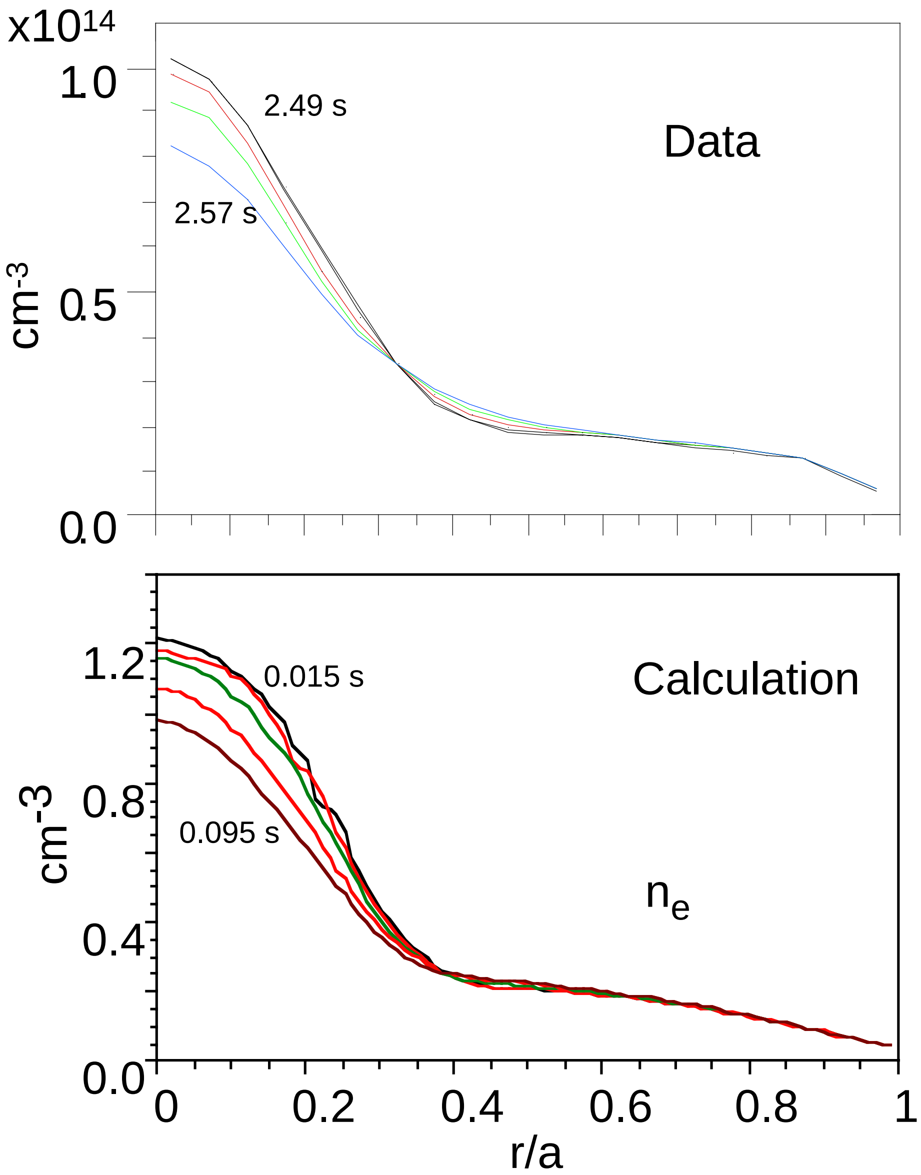
Forward and backward transition timescales

- **Experimental and computational transitional timescales display similar behavior. (from E. Synakowski)**



Evolution of density profiles in back transition

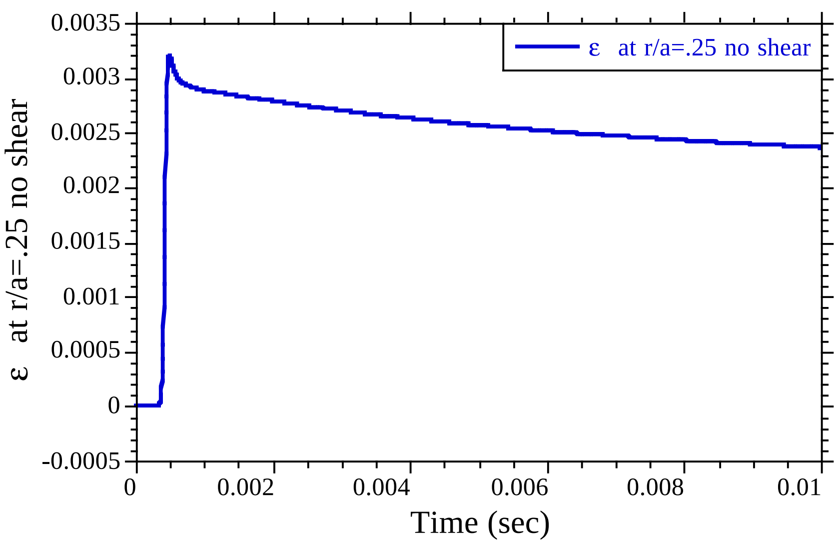
- **Experimental and computational profiles collapse in a similar manner**



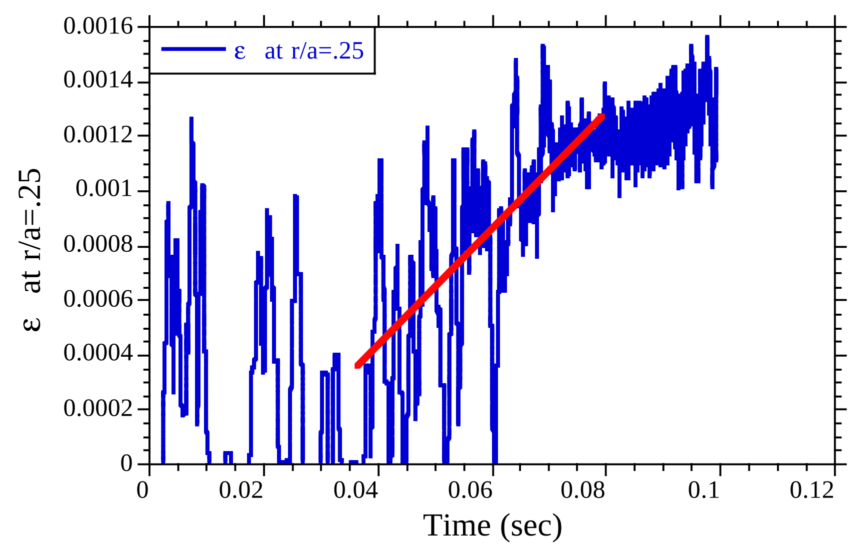
Shear term important in transition even below threshold

- Negative feedback through shear is term dominant effect in slowing back transition

Back transitions at 12 MW



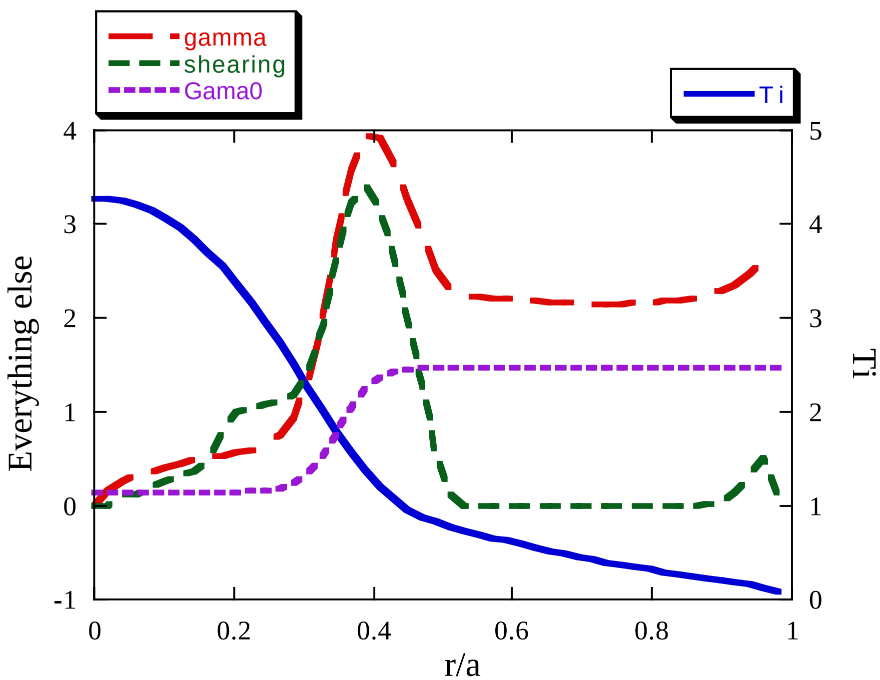
Back transition
(no shear effect)

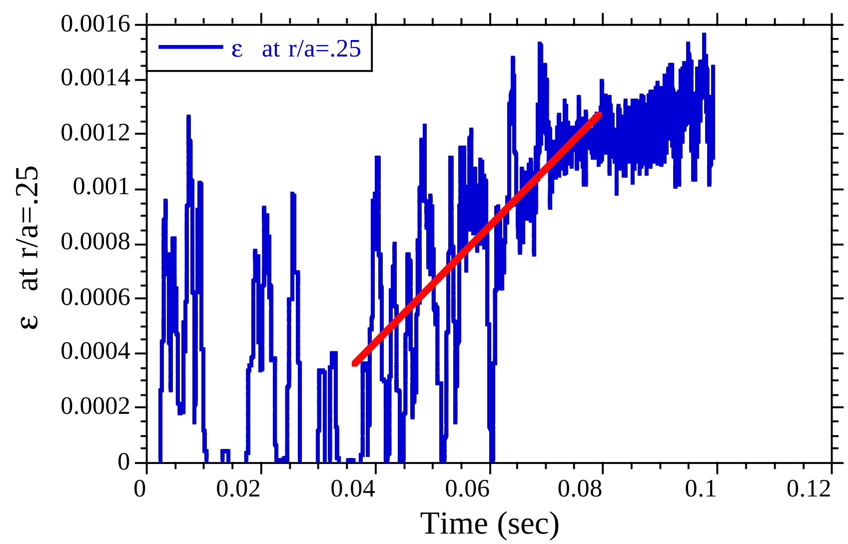


Back transition
(with shear effec

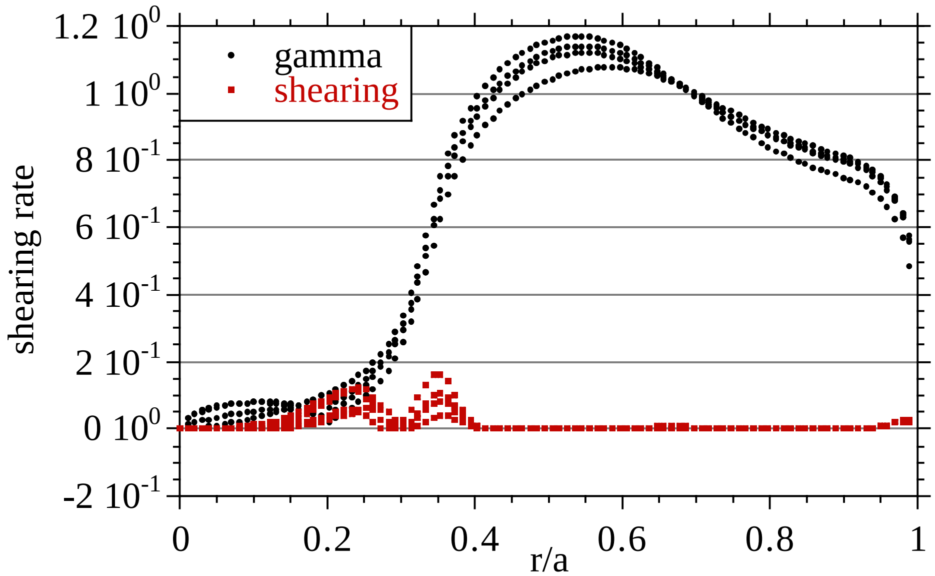
Bursty behavior of fluctuations

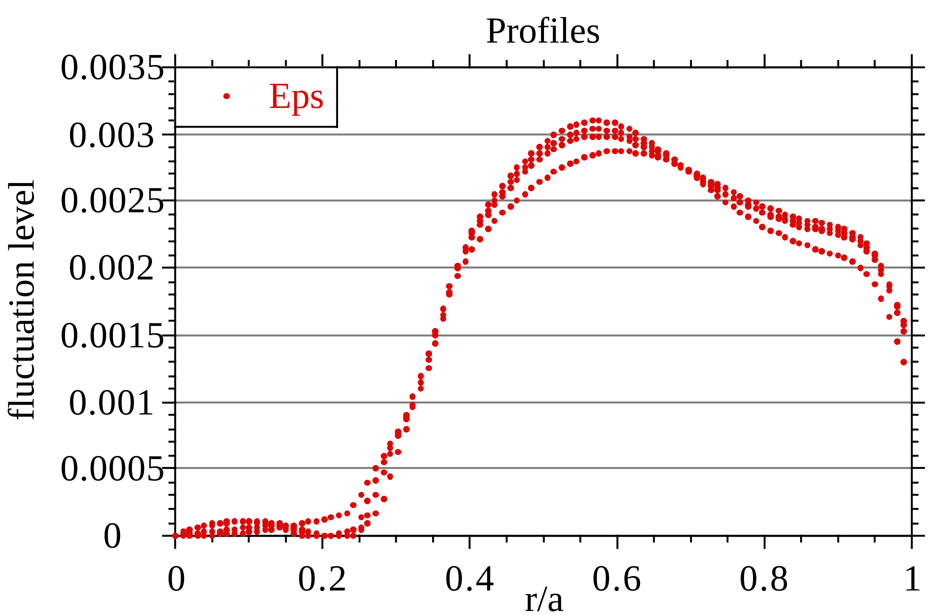
- **Bursts of fluctuations near threshold observed in both model and experiment**
- **Bursts may be attributed to relationship between profiles of γ and the shearing rate**



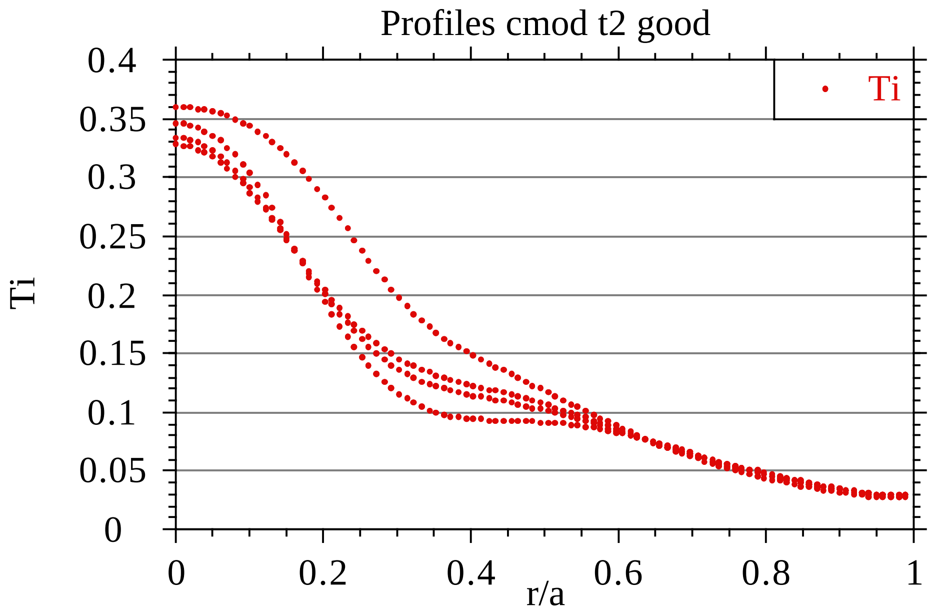


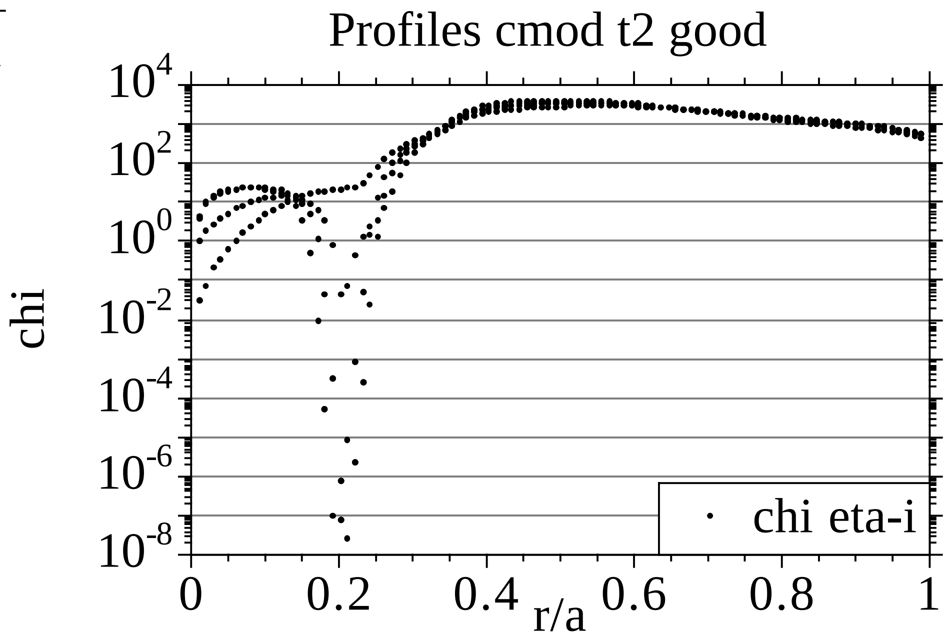
Transition triggered by pellet alone



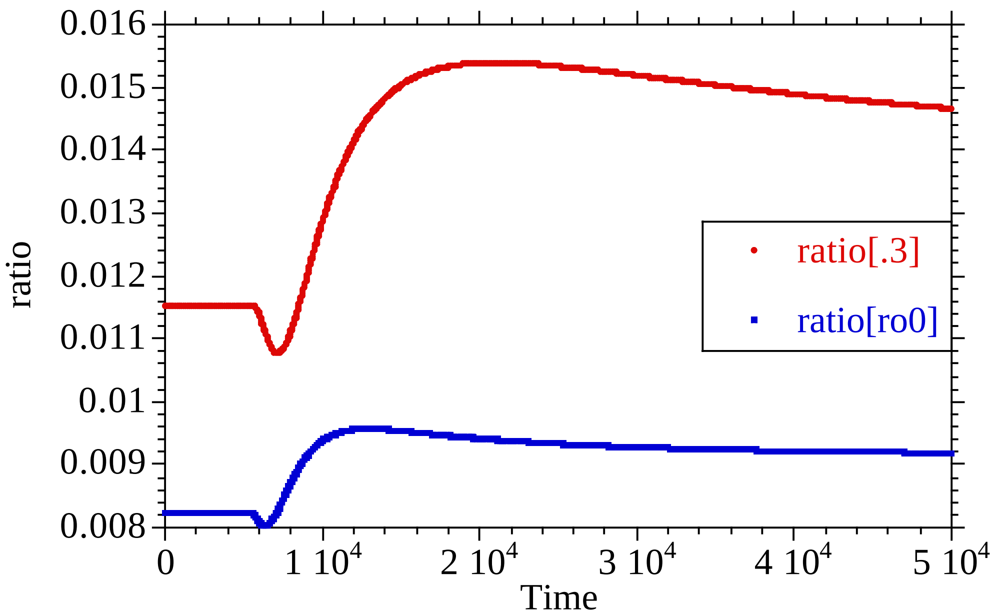


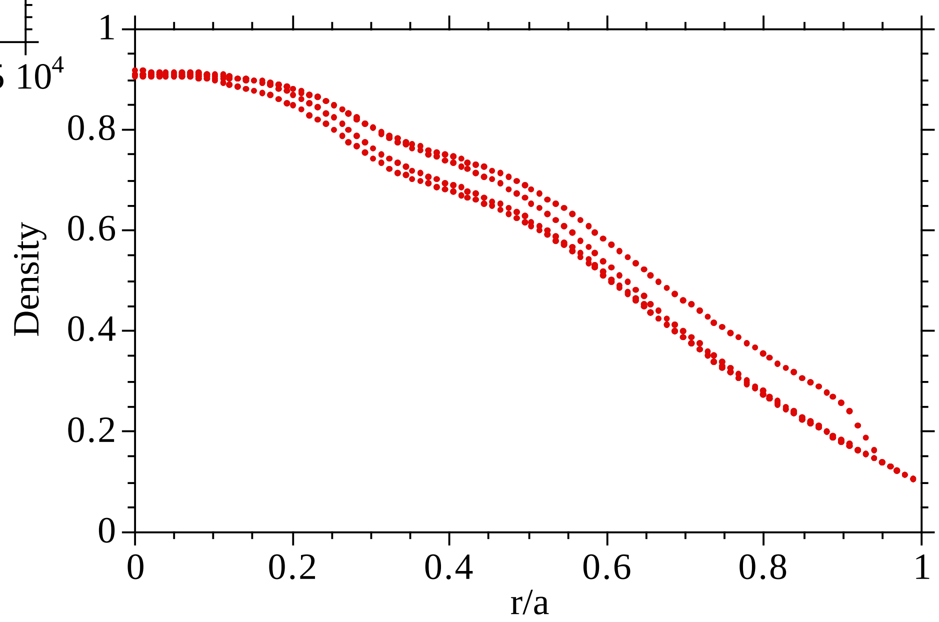
Pellet only transition





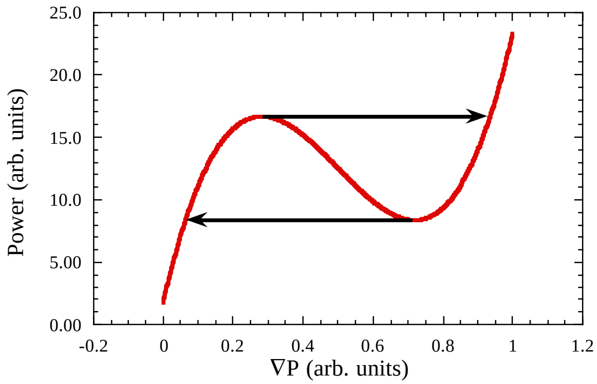
Internal barrier can be triggered by H-L transition





Trigger and control mechanisms

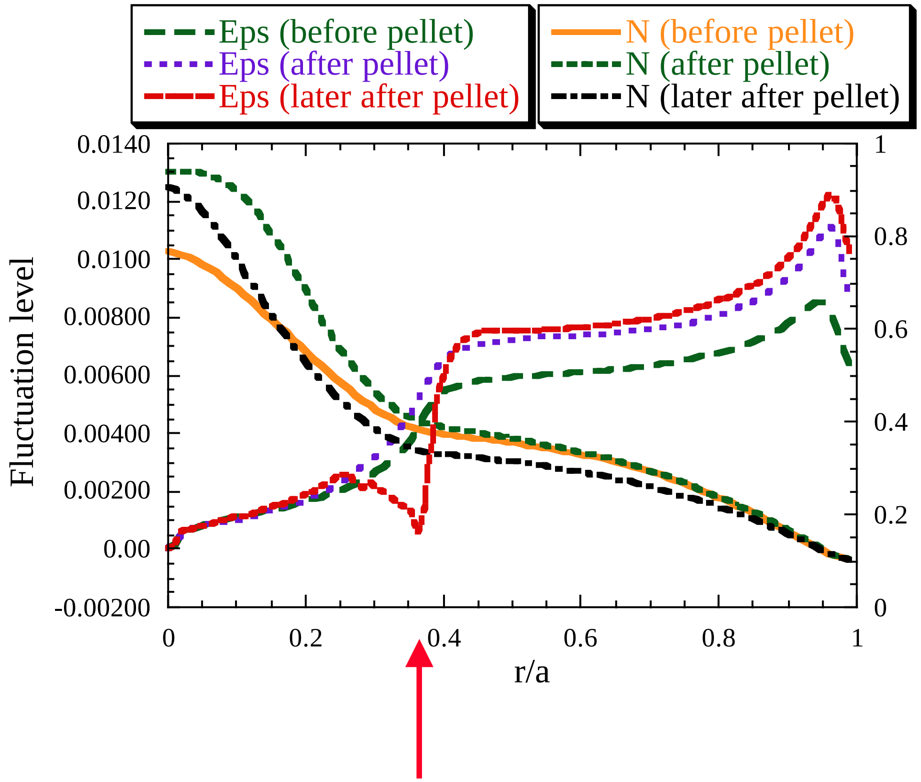
- **Utilizing the hysterisis in the system subcritical transitions may be possible with a transient supercritical event**
  - **any mechanism for transient flux (or gradient) pulse can trigger transition**
    - » **Driven sheared flows (V_θ or V_ϕ): beams, IBW etc.**
    - » **Heating**
    - » **Pellets or pellets and heating**
    - » **MHD**

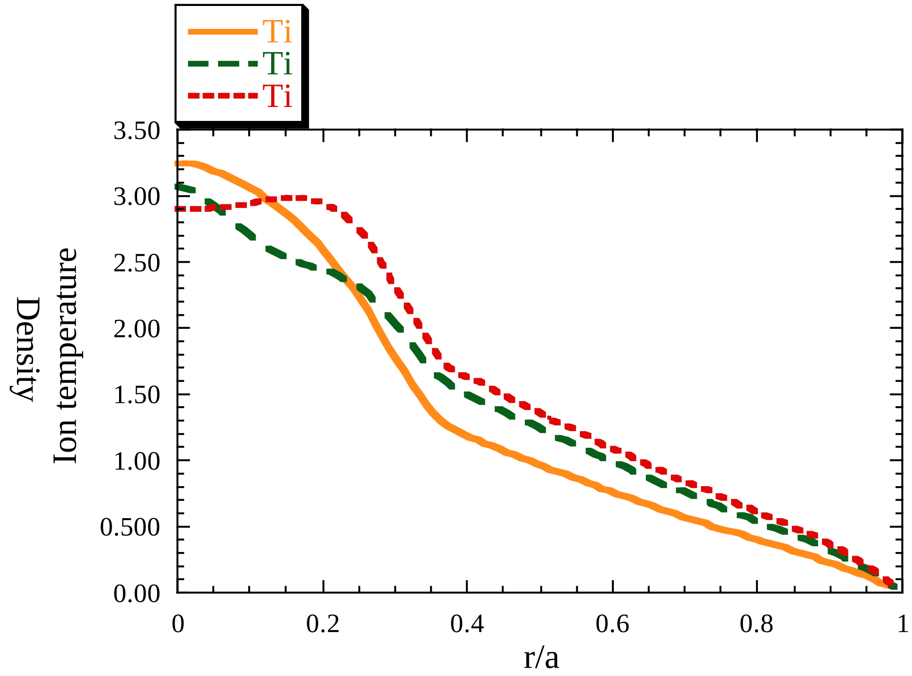


- **Control may be achieved through periodic releases of the stored energy**
    - » **Force back transitions with driven sheared flows (V_θ or V_ϕ) again beams, IBW etc.**
    - » **Moving the q profile through controlled off axis current drive**

Off axis heating and pellet can trigger sub-threshold transition

- **Combination of heating and pellet causes transition 4Mw below Pth - Hysterisis can then maintain it**

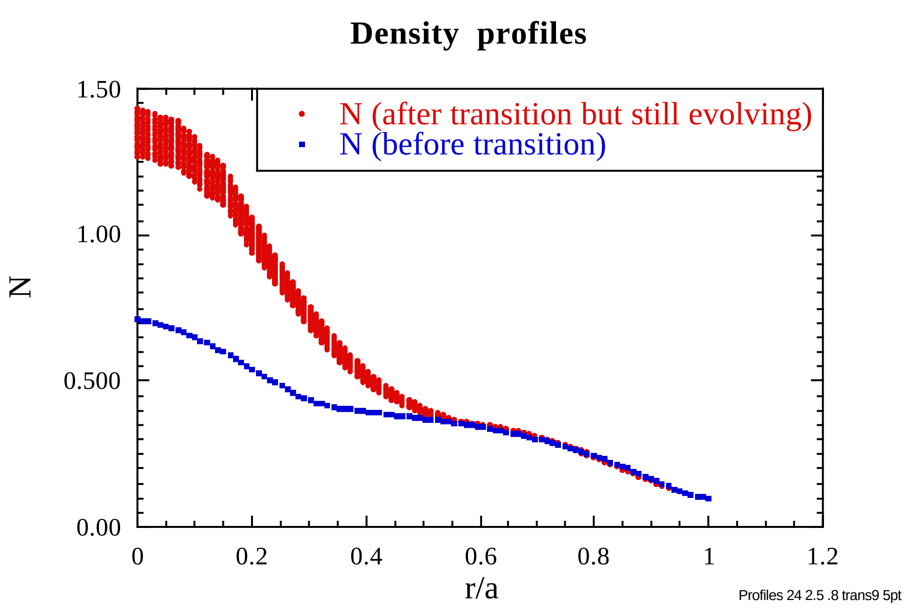


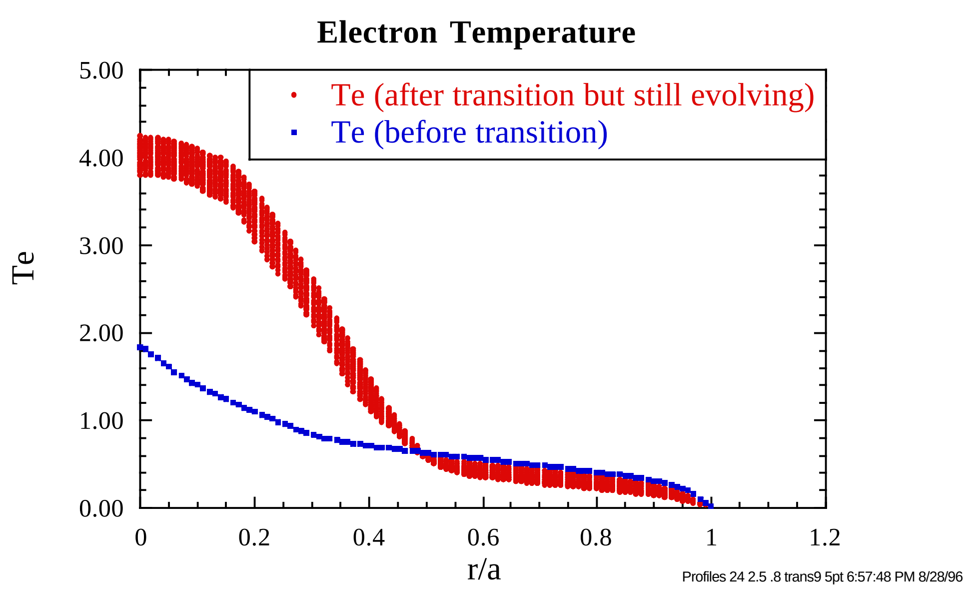


q_{min} and center of heating

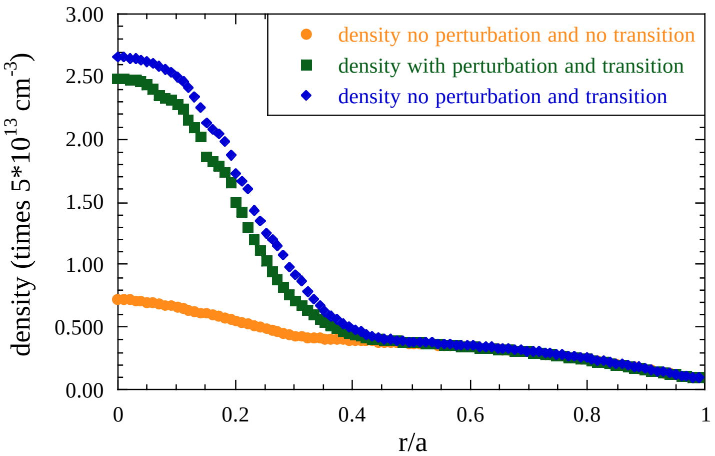
Control of profiles after transition

- **Without some mechanism for controlled release of particles and energy after the transition profiles run away (controlled by neoclassical diffusion)**
  - **profiles often end exceeding MHD stability threshold**

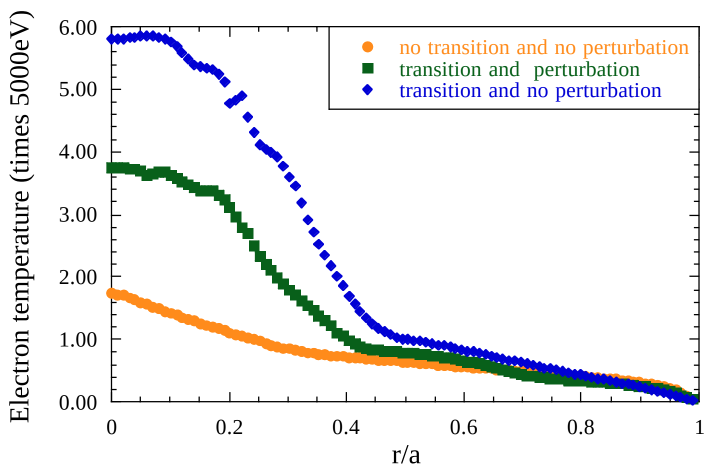


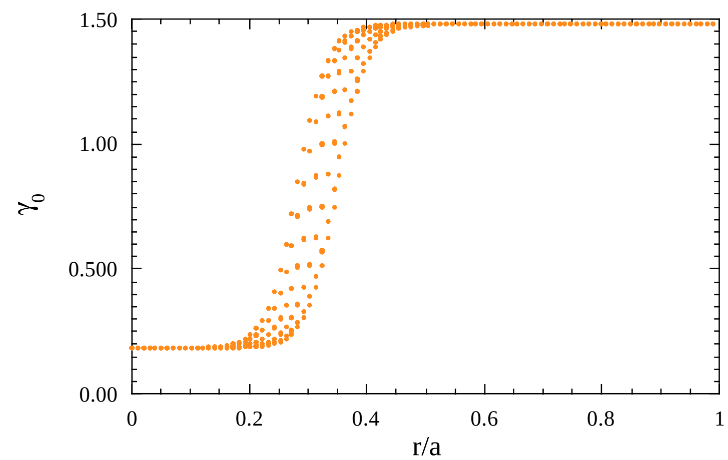


Small periodic changes in q profile can have a major effect on final state

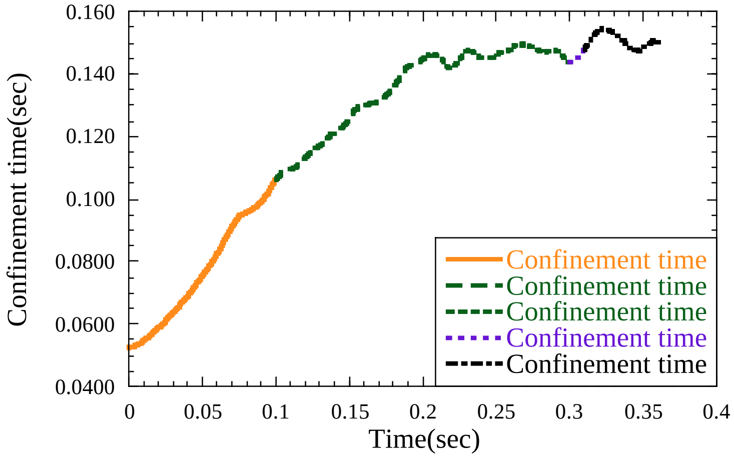


Perturbation reduces or stops outward propagation of barrier decreasing disruption probability





Confinement time reduced in perturbed case

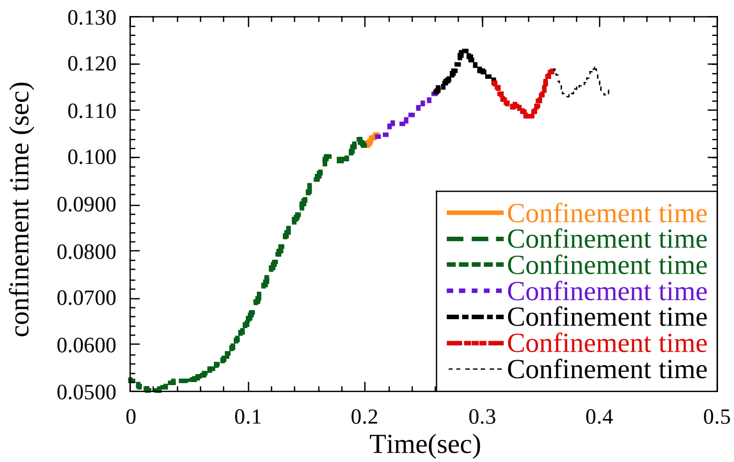


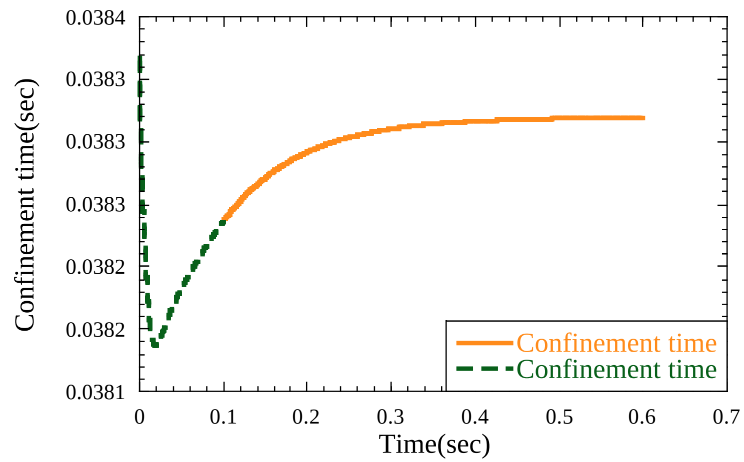
unperturbed

~30%reduction in t with 6% motion of q_{min} requiring < 8%change in current

perturbed

no transition





V_θ evolution

- **Recent experiments suggest that in ERS transition V_θ may play an important role in triggering the transition. It could also be used to force back transitions for accumulation control.**
  - **Can be an internal trigger or can help make super-critical region larger.**
  - **Could be driven directly by RF flow drive (IBW)**
  - **Reynolds stress is plausible mechanism for observed V_θ transition trigger**
  - **Can act to stop propagation of transition front.**

Adding V_θ evolution

- **Reynolds stress drive needs seed sheared flow to break symmetry and a gradient in the fluctuations for momentum transfer source**
  - **Given profiles, Reynolds stress drive is plausible mechanism for flow generation**

$$\frac{\partial \langle V_\theta \rangle}{\partial t} = -\mu \langle V_\theta \rangle + \alpha_3 \frac{1}{r^2} \frac{\partial}{\partial r}\left(r^2 \varepsilon \frac{\partial \varepsilon}{\partial r} \frac{\partial \langle V_E \rangle}{\partial r} \right) + D_0 \frac{1}{r} \frac{\partial}{\partial r}\left(r \varepsilon \frac{\partial \langle V_\theta \rangle}{\partial r} \right)$$

Viscous flow damping ($-\mu \langle V_\theta \rangle$)

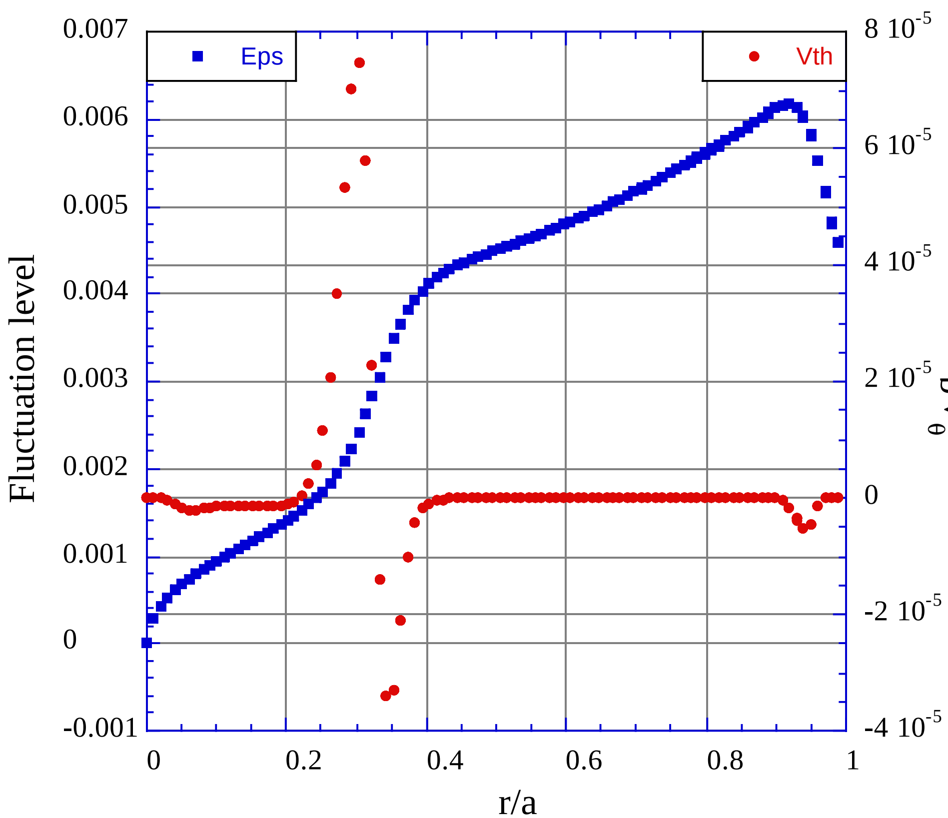
Turbulent Reynolds stress (α_3 term)

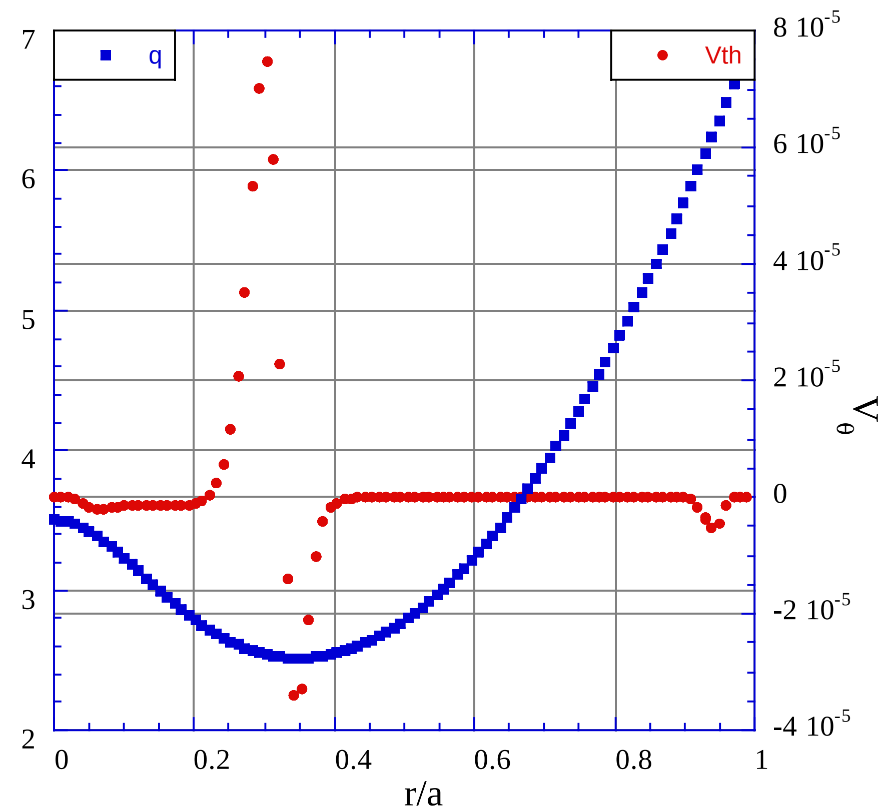
Turbulent diffusion (D_0 term)

Evolving profiles => Reynolds stress driven flow => Suppress turbulent transport => Steepen gradients => Trigger full pressure gradient driven transition => Kills fluctuations => Kills flow drive and flow

Localized shear flow is generated

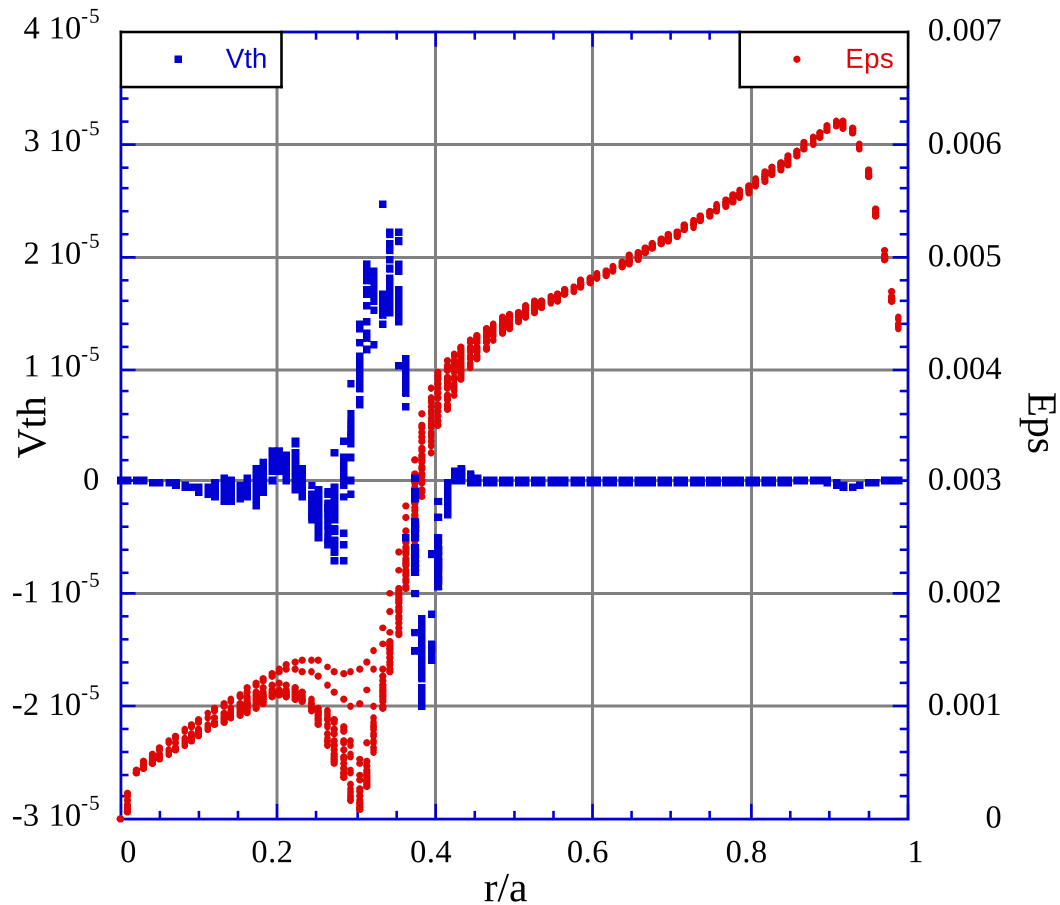
- **Up spike in V_θ adds to pressure gradient component of E_r**
- **Spike is localized just inside q_{min}**





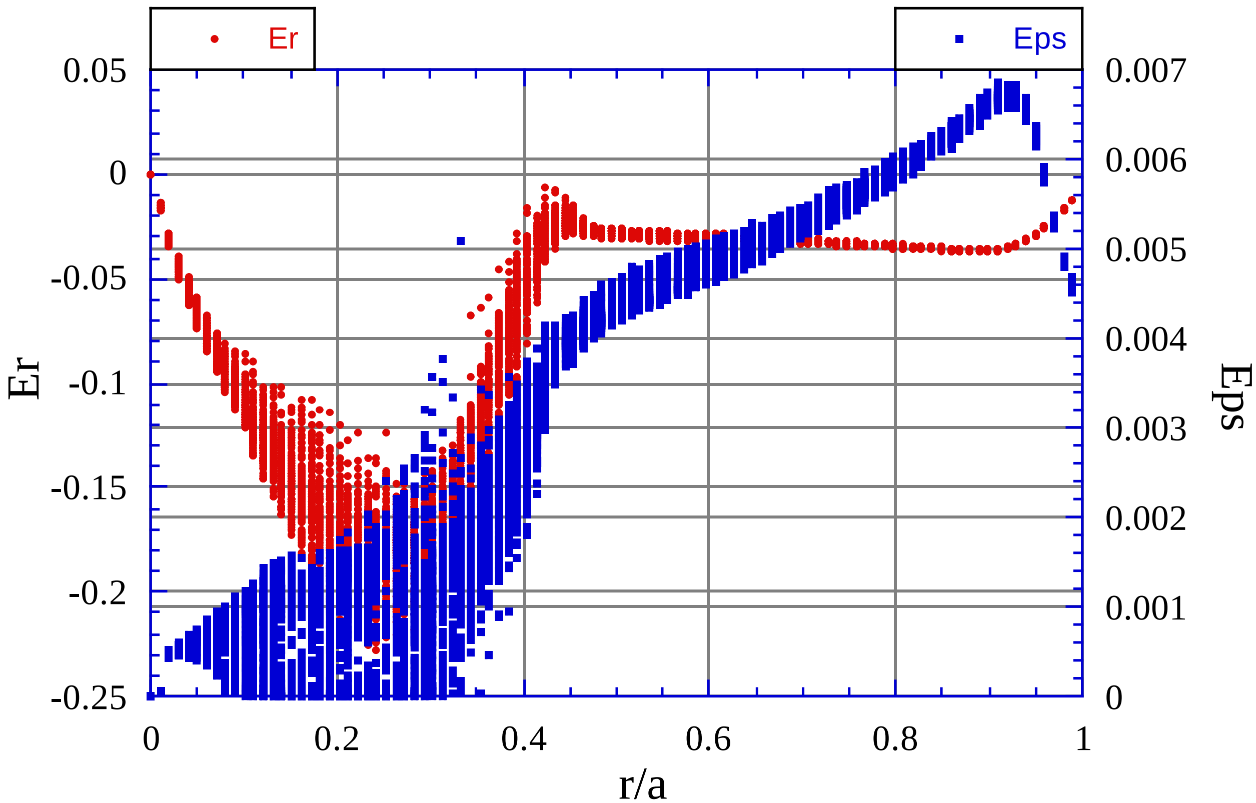
Sheared flow starts suppression

- **Flow can not fully suppress turbulence in steady state**



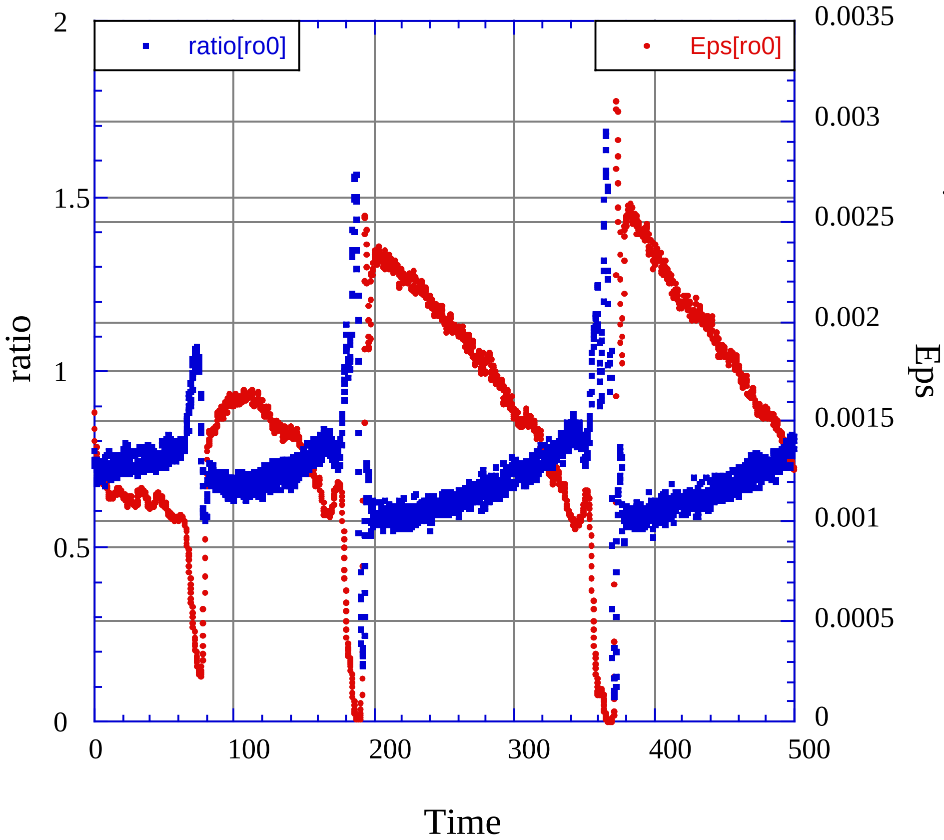
Pressure Gradient Finishes Transition

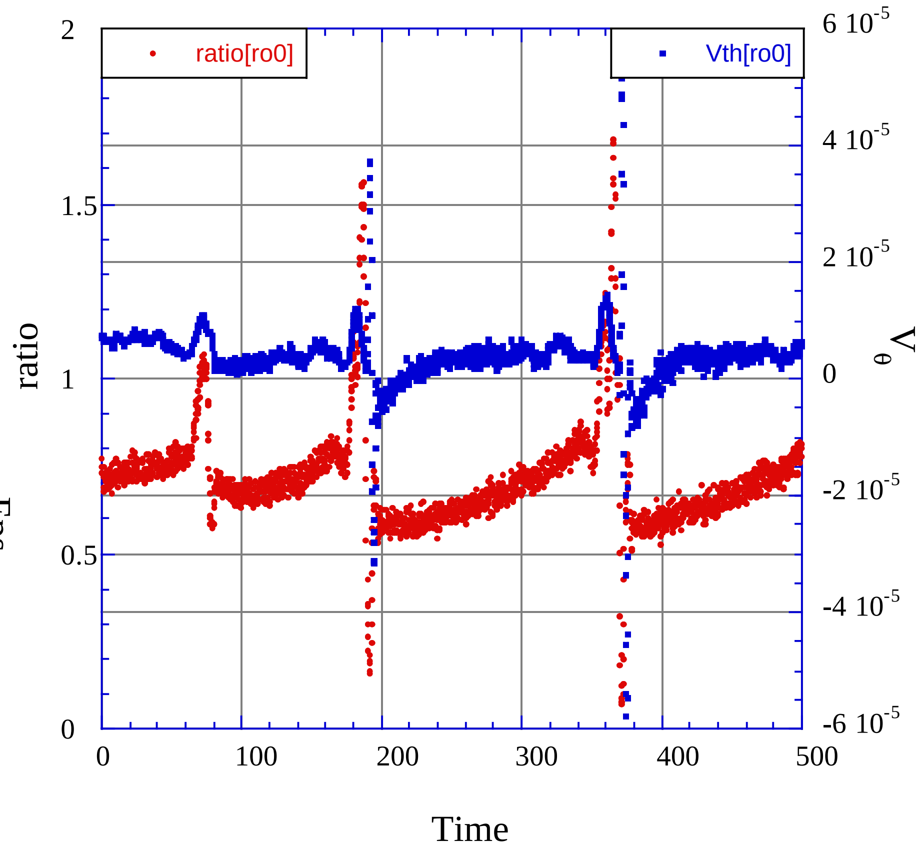
- **Transition is completed by pressure gradient growing and dominating the ExB flow shear, completing the transition and suppressing the fluctuations**



New rich bursting dynamics

- **Can exhibit preditor-prey like oscillations**





Transport model evolution equations in ASTRA

Grad-Shafranov equation is solved to account for magnetic geometry. Then transport equations for electron density, electron and ion pressure, poloidal magnetic flux (which gives current evolution) and toroidal momentum are evolved using the integral of the magnetic flux as radial coordinate.

$$\frac{\partial n_e}{\partial t} = -\nabla \cdot \left(D_n \nabla n_e \right) + S_{NBI} + S_{wall}$$

$$\frac{3}{2}\frac{\partial \left(n_e T_e \right)}{\partial t} = -\nabla \cdot \left(\chi_e n_e \nabla T_e + \frac{5}{2} D_n T_e \nabla n_e \right) - \frac{\Gamma_n}{n_e} \nabla \left(n_e T_i \right) - P_{ei} + P_{ohm} + P_{NBI,e}$$

$$\frac{3}{2}\frac{\partial \left(n_i T_i \right)}{\partial t} = -\nabla \cdot \left(\chi_i n_i \nabla T_i + \frac{5}{2} D_n T_i \nabla n_i \right) + \frac{\Gamma_n}{n_e} \nabla \left(n_e T_i \right) + P_{ei} + P_{NBI,i}$$

$$q B_\theta \sigma \frac{\partial \psi}{\partial t} = \left\langle J_{\parallel} B \right\rangle + \left\langle \left(\mathbf{J}_{\mathbf{NBI}} + \mathbf{J}_b \right) \cdot \mathbf{B} \right\rangle$$

$$\frac{\partial M}{\partial t} = -\nabla \cdot \left(D_M \nabla M \right) + S_M$$

Plus Fluctuation level and Radial Electric field

Instability drive

Non linear damping (saturation term)

Sheared E_r Field stabilization (suppression

$$\frac{\partial \varepsilon}{\partial t} = \left[\gamma_{\eta_i} - \alpha_1 \varepsilon - \alpha_2 \frac{\omega_s^{\ 2}}{\gamma_{\eta_i}} \right] \varepsilon + \nabla \left(D_\varepsilon \nabla \varepsilon \right); \qquad \varepsilon \equiv \left| \frac{\tilde{n}_k}{n_0} \right|^2$$

– **The linear growth rate and the E_r shearing have different dependencies on the thermodynamic profiles. Where $\sqrt{(\alpha_2)}\,\omega_s \sim \gamma$, the only stable solution for fluctuations is $\varepsilon = 0$.**

$$\begin{cases} \gamma_{\eta_i} \sim (\nabla P)^{1/2} \\ \omega_s \sim E_r' \end{cases}$$

$$E_r = \frac{1}{z_i e n} \nabla P + V_\theta B_\varphi - V_\varphi B_\theta$$

The shear of any of these three terms can cause the local suppression of fluctuations.

Transport coefficients

General form: $$D = D_0 + D_A + D_{RB}$$

Background neoclassical level

Core η_i anomalous transport ε^2
[Biglari, Diamond, Rosenbluth]

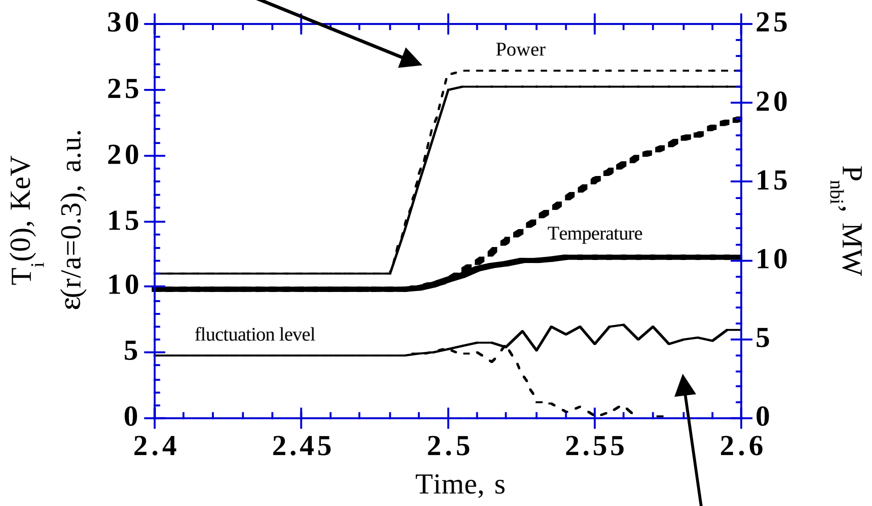
Edge resistive ballooning
[Guzdar, Drake]

Exception: neoclassical particle convection [Hirshman, Sigmar] for bootstrap current:

$$J_B \sim C_n \frac{n_e'}{n_e} + C_e \frac{T_e'}{T_e} + C_i \frac{T_i'}{T_i}$$

Transition using NBI power

Slight differences in control parameters (NBI power in this case) drive the system to totally differing states

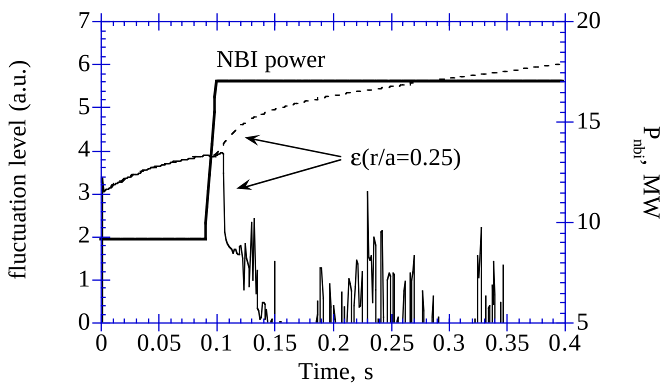


Non-transitioned state is characterized, due to the proximity to the threshold, by bursts in the fluctuation level

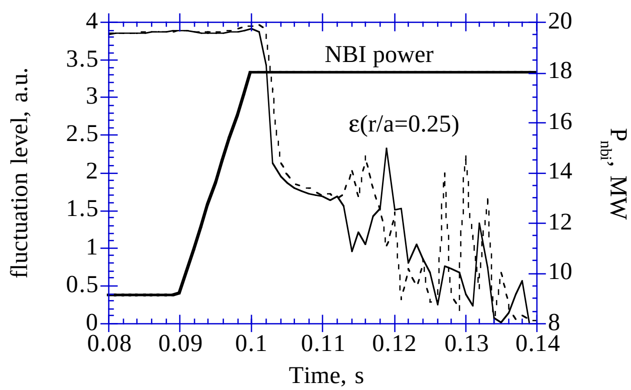
Effect of current evolution on transition

RS state is identically imposed at startup (analogous to previous example) in the next four cases up to t=0.09 s. Then the power is ramped from t=0.09 to t=0.10 s in calculations with (dashes) and without (solid) current evolution effect on growth rate:

NBI power to 17 MW



NBI power to 18 MW

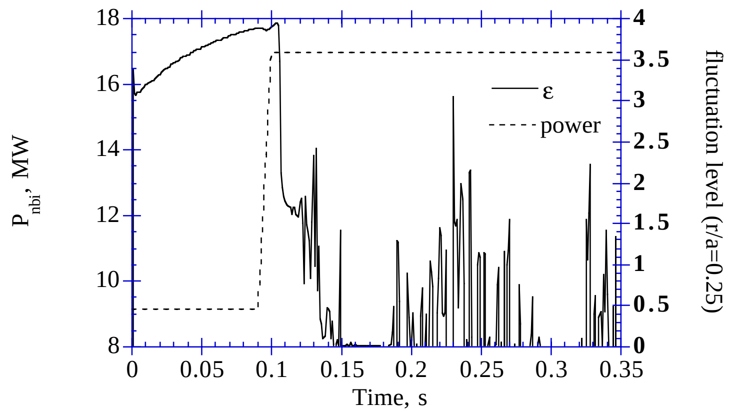


When P_{NBI}~Pth, the modification of q-profile can change Pth. In this case, the evolution of the current avoids the transition

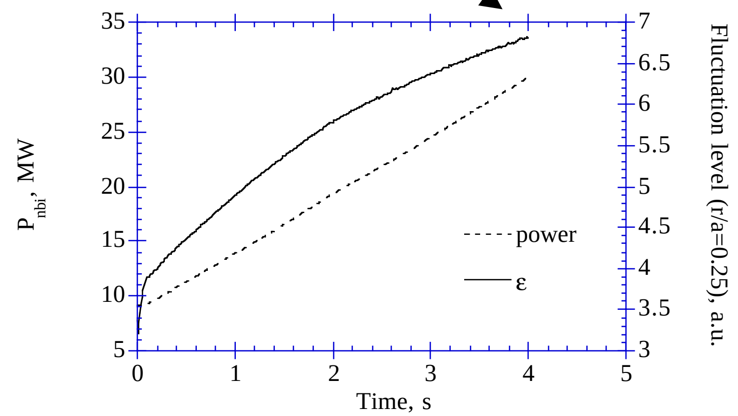
When P_{NBI}>Pth, the time scales associated with the evolution of the turbulence (much faster than current diffusion scales) make both cases very similar

Effect of power ramping rate on transition

Suppression of fluctuations not seen here because the transition drives the system beyond MHD equilibrium



(a)



(b)

Current profile is held constant from t=0.09 s. When the power is suddenly increased (a), the transition is reached at P_{NBI}=18 MW. If, instead, the power is slowly ramped (b), the threshold is not reached until P_{NBI}=34 MW. The overshoot of ion temperature right after the power jump can help trigger the transition. But, a strong perturbation of the gradient/curvature of the pressure profile might be responsible as well.

Summary

- Minimal model for ERS/NCS mode gives q(r) modulated Er shear induced transport barrier
- Strong negative shear unnecessary lower γ -> lower threshold
  - Gradient in γ gives boundary turbulence within quenched (unlike edge transition)
- Suggests optimization of deposition profile relative to shear profile (both Er and q)
- A variety of triggering mechanisms can lead to the same transition dynamics
  - increased power, H - L transitions, pellets, MHD activity, transient heating, change in heating profile, increased edge radiation (cold pulses), V_ϕ, V_θ
- Can be triggered by Reynolds stress driven flow
- MHD stability and it's feedback can be calculated via coupling to full ideal/resistive MHD code (FAR)