Dynamic Modeling of Transport Barrier Formation and Evolution

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Minimal models with radial profile of growth rates display:
1) Critical transitions with bursting behavior and multiple time scales in the dynamics
2) Similar transition dynamics from a wide variety of trigger mechanisms
3) Importance of deposition profiles
4) Reynolds stress driven flows triggering transition

==> Dynamics of transition suggests control strategies
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Outline

• Motivation
• Basic model
  – Fluctuation and Pressure gradient evolution
• Extended transport/transition model
  – Basic model
  – Transition dynamics
  – Deposition profile effects
  – Reynolds stress driven poloidal flow
  – Methods for initiation
  – Methods for control
• Summary
Motivation

- Transitions from a reversed/weak shear mode to a high confinement (ERS, ENCS etc) mode observed in different devices suggests underlying details of initiation may differ, dynamics of transition are the same.

- What questions can a simple dynamical model answer?

- Challenge to find:
  - threshold
    »  threshold quantity(ies) for control (opportunity for integrated approach)
  - transition dynamics
  - barrier width and localization
  - mechanism for post-transition control of gradients
  - role of magnetic shear in transition and sustainment
Heuristics of Model

- Radially inhomogeneous turbulent growth rates (due to magnetic shear profile) and deposition profiles can produce sheared ExB flows.

- Sheared flows increase decorrelation of turbulence $\Rightarrow$ reducing the turbulent amplitude and transport.

- Coupling exists between the turbulent fluctuations and the pressure gradient which can also feed back on the other sheared flows ($V_\theta$ and $V_\phi$).
Shear Flows Effect on Transport

4 mechanisms for reduction of turbulent transport

- Shear modification of linear drive (plasmas +)
- Shear decorrelation of turbulence (all turbulent systems)
- Phase shift between advected and advecting quantities
- Shear decorrelation of transport events (avalanches)

Linear Drive $\propto$ Frequency
Shear Flow can change the Freq. and therefore change the drive

Turbulent Amplitude $\propto$ Drive

Turbulent Transport $\propto$ Turbulent Amplitude
Coupled nonlinear envelope equations for the fluctuations level and pressure gradient

\[
\frac{\partial E}{\partial t} = \gamma_0 NE - \alpha_1 E^2 - \alpha_2 \langle V'_E \rangle^2 E + D_0 \frac{\partial}{\partial x} \left( E \frac{\partial E}{\partial x} \right)
\]

\[
\frac{\partial N}{\partial t} = -\alpha_4 N + Q + D_0 \frac{\partial}{\partial x} \left( E \frac{\partial N}{\partial x} \right)
\]

where

\[
E \equiv \left| \frac{\tilde{n}_k}{n_0} \right|^2 \\
V'_E = V'_\theta - \beta V'_\Phi - \alpha N^2 \\
N \equiv -\frac{a}{\langle P \rangle} \frac{dP}{dr}
\]
More complete transport model with transition model provides reasonable thresholds and same dynamics

- **Transport model:**

\[
\frac{\partial n}{\partial t} = S_{NBI} + S_{gp} + \frac{1}{r} \frac{\partial}{\partial r} \left[ r D_n \frac{\partial n}{\partial r} \right]
\]

\[
\frac{3}{2} \frac{\partial n T_i}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left[ r \left( \chi_i n \frac{\partial T_i}{\partial r} + \frac{5}{2} D_n T_i \frac{\partial n}{\partial r} \right) \right] - D_n \frac{1}{n} \frac{\partial n}{\partial r} \frac{\partial n T_i}{\partial r} + Q_{NBI}^i + Q_{ei}(T_e - T_i)
\]

\[
\frac{3}{2} \frac{\partial n T_e}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left[ r \left( \chi_e n \frac{\partial T_e}{\partial r} + \frac{5}{2} D_n T_e \frac{\partial n}{\partial r} \right) \right] + D_n \frac{1}{n} \frac{\partial n}{\partial r} \frac{\partial n T_i}{\partial r} + Q_{NBI}^e + Q_{Ohm} + Q_{ie}(T_e - T_i)
\]

- **System is closed with a Fluctuation envelope equation**

\[
\frac{\partial \varepsilon}{\partial t} = \gamma - \alpha_1 \varepsilon - \alpha_2 \left[ \frac{r}{q} \frac{\partial}{\partial r} \left( \frac{q E_r}{r B_\phi} \right) \right]^2 \varepsilon + \frac{1}{r} \frac{\partial}{\partial r} \left[ r D_\varepsilon \frac{\partial \varepsilon}{\partial r} \right]
\]

and the equation for \(E_r\)

\[
E_r = -V_\theta + V_\phi \frac{B_\theta}{B_0} + \alpha \left[ \frac{\partial T}{\partial r} + \frac{T}{N} \frac{\partial N}{\partial r} \right]
\]
Additional dependencies

- **Growth rate and diffusivities as functions of plasma parameters**

\[
\gamma_{\eta_i} = k_\theta \rho_s \frac{c_s}{a} f(\hat{S}) \left( \frac{a}{R} \right)^{1/2} \left( \frac{a}{L_n} + \frac{a}{L_T} \right)^{1/2} \left( \frac{T_i}{T_e} \right)^{1/2}
\]

\[
\chi_{in_i} = k_\theta \rho_s c_s a \left( \frac{a}{R} \right)^{1/2} \left( \frac{a}{L_n} + \frac{a}{L_T} \right)^{-1/2} \left( \frac{T_i}{T_e} \right)^{1/2} \chi_{in_i}
\]

\[
\chi_{en_i} = D_{\eta_i} = k_\theta \rho_s \frac{c_s}{a v_e} \left( \frac{a^2}{L_n R} \right)^{1/2} \left( 1 + \frac{L_n}{L_T} \right)^{1/2} \left( \frac{T_i}{T_e} \right)^{1/2} \chi_{in_i}
\]

with

\[
L_n = n(dn/dr)^{-1} \quad L_T = T_i(dT_i/dr)^{-1} \quad \alpha_2 = \left( \frac{\Delta_r}{r \Delta_{\theta}} \right)^2 \frac{1}{\gamma}
\]
Initiation of transition occurs due to combination of $q$, $\gamma$ and deposition profiles

- **Shear parameter:**
  
  \[
  \text{Shear} \approx -\frac{r}{q} \frac{\partial}{\partial r} \left( \frac{qE_r}{B_\psi r} \right) \quad \text{(see Hahm and Burrell '95)}
  \]

- **Need low growth rate, steep gradient (i.e. edge of deposition region) and appropriate $q$ profiles**

**Before**

**After**
The effect of q profiles on the shearing term

- **Shear in the q profile can both move and increase the peak in the shearing rate**

- **Change in q profile can initiate transition in different location**
Profile evolution during transition

- Fluctuations are quenched
  - transiently reduced outside transition region due to decreased flux

- Temperature and density profiles grow and steepen in quenched region
Transition can be seen in confinement time at $P_{th}$.

- **Transition occurs when ratio is 1 giving a confinement bifurcation through suppression of the fluctuation driven transport.**

![Graph showing the relationship between ratio and confinement time.](image-url)
Flux through surface dependent on deposition width

- Threshold dependent on local gradient => flux through surface by
  \(\nabla P = \Gamma/D\) => power/particles deposited inside surface

Threshold has strong dependence on deposition profile width

Possible explanation for change in TFTR power (Synakowski et al PRL 97) threshold going from D to T
Initiation position can move with deposition profile

- Transition can initiate well inside $q_{\text{min}}$ for narrow deposition profiles
Deposition profiles important for location and extent of transitioned region

- Same total power but different profile widths give very different flux through qmin surface
Full transition or localized transition dependent on deposition profile

- Broader or shifted deposition profile leads to narrow transitioned region (related to type II JT-60U ITB?)
- Narrower or better centered deposition profiles lead to broader ITB (related to type I JT-60U ITB?)
• Narrow region of fluctuations suppressed for broad deposition profile
• Broader region for narrower deposition profile
Footpoint moves out during transition

- Steep gradient region can move passed $q_{\text{min}}$ allowing disruptions

**24MW transition**

Density evolution during forward transition

![Graph showing density evolution](image)
Forward transition much faster than back transition

- Forward transition has positive feedback through gradient so bookstraps itself up
- Backtransition has negative feedback through increased flux due to increased diffusivities => this with growth rate effects leads to slower transitions

Forward and back transition for 12 MW
Forward and backward transition timescales

- Experimental and computational transitional timescales display similar behavior. (from E. Synakowski)
Evolution of density profiles in back transition

- Experimental and computational profiles collapse in a similar manner
Shear term important in transition even below threshold

• Negative feedback through shear term dominant effect in slowing back transition

Back transitions at 12 MW

[Graphs showing back transitions at 12 MW, one with no shear effect and another with shear effect.]
Bursty behavior of fluctuations

- Bursts of fluctuations near threshold observed in both model and experiment
- Bursts may be attributed to relationship between profiles of $\gamma$ and the shearing rate
Transition triggered by pellet alone
Pellet only transition

Profiles cmod t2 good

Ti

r/a

chi

Profiles cmod t2 good

Chi eta-i

r/a
Internal barrier can be triggered by H-L transition
Trigger and control mechanisms

- Utilizing the hysteresis in the system subcritical transitions may be possible with a transient supercritical event
  - any mechanism for transient flux (or gradient) pulse can trigger transition
    » Driven sheared flows ($V_\theta$ or $V_\phi$): beams, IBW etc.
    » Heating
    » Pellets or pellets and heating
    » MHD

- Control may be achieved through periodic releases of the stored energy
  » Force back transitions with driven sheared flows ($V_\theta$ or $V_\phi$) again beams, IBW etc.
  » Moving the $q$ profile through controlled off axis current drive
Off axis heating and pellet can trigger sub-threshold transition

- Combination of heating and pellet causes transition 4Mw below Pth - Hysterisis can then maintain it

\[ q_{\text{min}} \text{ and center of heating} \]
Control of profiles after transition

- Without some mechanism for controlled release of particles and energy after the transition profiles run away (controlled by neoclassical diffusion)
  - profiles often end exceeding MHD stability threshold
Small periodic changes in q profile can have a major effect on final state

Perturbation reduces or stops outward propagation of barrier decreasing disruption probability
Confinement time reduced in perturbed case

- unperturbed
  ~30% reduction in $t$ with 6% motion of $q_{\text{min}}$ requiring < 8% change in current

- perturbed
  no transition
Recent experiments suggest that in ERS transition $V_\theta$ may play an important role in triggering the transition. It could also be used to force back transitions for accumulation control.

- Can be an internal trigger or can help make super-critical region larger.
- Could be driven directly by RF flow drive (IBW)
- Reynolds stress is plausible mechanism for observed $V_\theta$ transition trigger
- Can act to stop propagation of transition front.
Adding $V_\theta$ evolution

- Reynolds stress drive needs seed sheared flow to break symmetry and a gradient in the fluctuations for momentum transfer source
  - Given profiles, Reynolds stress drive is plausible mechanism for flow generation

\[
\frac{\partial \langle V_\theta \rangle}{\partial t} = -\mu \langle V_\theta \rangle + \alpha_3 \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \varepsilon \frac{\partial}{\partial r} \frac{\partial \langle V_E \rangle}{\partial r} \right) + D_0 \frac{1}{r} \frac{\partial}{\partial r} \left( r\varepsilon \frac{\partial \langle V_\theta \rangle}{\partial r} \right)
\]

Viscous flow damping
Turbulent Reynolds stress
Turbulent diffusion

Evolving profiles => Reynolds stress driven flow =>
Suppress turbulent transport => Steepen gradients =>
Trigger full pressure gradient driven transition =>
Kills fluctuations => Kills flow drive and flow
Localized shear flow is generated

- Up spike in $V_\theta$ adds to pressure gradient component of $E_r$
- Spike is localized just inside $q_{\text{min}}$
Sheared flow starts suppression

• Flow cannot fully suppress turbulence in steady state
Pressure Gradient Finishes Transition

- Transition is completed by pressure gradient growing and dominating the ExB flow shear, completing the transition and suppressing the fluctuations.
New rich bursting dynamics

- Can exhibit predator-prey like oscillations
Grad-Shafranov equation is solved to account for magnetic geometry. Then transport equations for electron density, electron and ion pressure, poloidal magnetic flux (which gives current evolution) and toroidal momentum are evolved using the integral of the magnetic flux as radial coordinate.

\[
\frac{\partial n_e}{\partial t} = -\nabla \cdot \left( D_{n_e} \nabla n_e \right) + S_{\text{NBI}} + S_{\text{wall}}
\]

\[
\frac{3}{2} \frac{\partial (n_e T_e)}{\partial t} = -\nabla \cdot \left( \chi_e n_e \nabla T_e + \frac{5}{2} D_{n_e T_e} \nabla n_e \right) - \frac{\Gamma_n}{n_e} \nabla \left( n_e T_i \right) - P_{ei} + P_{\text{ohm}} + P_{\text{NBI},e}
\]

\[
\frac{3}{2} \frac{\partial (n_i T_i)}{\partial t} = -\nabla \cdot \left( \chi_i n_i \nabla T_i + \frac{5}{2} D_{n_i T_i} \nabla n_i \right) + \frac{\Gamma_n}{n_e} \nabla \left( n_e T_i \right) + P_{ei} + P_{\text{NBI},i}
\]

\[
qB_\theta \sigma \frac{\partial \psi}{\partial t} = \langle J || B \rangle + \left\langle (J_{\text{NBI}} + J_b) \cdot B \right\rangle
\]

\[
\frac{\partial M}{\partial t} = -\nabla \cdot \left( D_M \nabla M \right) + S_M
\]
The linear growth rate and the $E_r$ shearing have different dependencies on the thermodynamic profiles. Where $\sqrt{\alpha_2} \omega_s \sim \gamma$, the only stable solution for fluctuations is $\varepsilon = 0$.

For any of these three terms, the shear can cause the local suppression of fluctuations.

$$
\frac{d\varepsilon}{dt} = \left[ \gamma \eta_i - \alpha_1 \varepsilon - \alpha_2 \frac{\omega_s^2}{\gamma \eta_i} \right] \varepsilon + \nabla(D \nabla \varepsilon);
$$

$$
\varepsilon \equiv \left| \frac{\tilde{n}_k}{n_0} \right|^2
$$

$$
E_r = \frac{1}{z_i e n} \nabla P + V_\theta B_\varphi - V_\varphi B_\theta
$$
Transport coefficients

General form:

\[ D = D_0 + D_A + D_{RB} \]

Background neoclassical level
Core \( \eta_1 \) anomalous transport \( \eta^2 \) [Biglari, Diamond, Rosenbluth]
Edge resistive ballooning [Guzdar, Drake]

Exception: neoclassical particle convection [Hirshman, Sigmar] for bootstrap current:

\[ J_B \sim C_n \frac{n_e'}{n_e} + C_e \frac{T_e'}{T_e} + C_i \frac{T_i'}{T_i} \]
Slight differences in control parameters (NBI power in this case) drive the system to totally differing states.

Non-transitioned state is characterized, due to the proximity to the threshold, by bursts in the fluctuation level.
Effect of current evolution on transition

RS state is identically imposed at startup (analogous to previous example) in the next four cases up to \( t = 0.09 \) s. Then the power is ramped from \( t = 0.09 \) to \( t = 0.10 \) s in calculations with (dashes) and without (solid) current evolution effect on growth rate:

When \( P_{\text{NBI}} \sim P_{\text{th}} \), the modification of q-profile can change \( P_{\text{th}} \). In this case, the evolution of the current avoids the transition

When \( P_{\text{NBI}} > P_{\text{th}} \), the time scales associated with the evolution of the turbulence (much faster than current diffusion scales) make both cases very similar
Effect of power ramping rate on transition

Suppression of fluctuations not seen here because the transition drives the system beyond MHD equilibrium

Current profile is held constant from \( t=0.09 \) s. When the power is suddenly increased (a), the transition is reached at \( P_{\text{NBI}}=18 \) MW. If, instead, the power is slowly ramped (b), the threshold is not reached until \( P_{\text{NBI}}=34 \) MW. The overshoot of ion temperature right after the power jump can help trigger the transition. But, a strong perturbation of the gradient/curvature of the pressure profile might be responsible as well.
Summary

• Minimal model for ERS/NCS mode gives \( q(r) \) modulated \( \text{Er} \) shear induced transport barrier

• Strong negative shear unnecessary lower \( \gamma \) -> lower threshold
  – Gradient in \( \gamma \) gives boundary turbulence within quenched (unlike edge transition)

• Suggests optimization of deposition profile relative to shear profile (both \( \text{Er} \) and \( q \))

• A variety of triggering mechanisms can lead to the same transition dynamics
  – increased power, H - L transitions, pellets, MHD activity, transient heating, change in heating profile, increased edge radiation (cold pulses), \( \dot{V}_\phi, \dot{V}_\theta \)

• Can be triggered by Reynolds stress driven flow

• MHD stability and it’s feedback can be calculated via coupling to full ideal/resistive MHD code (FAR)