# Why the Rhine's length is crucially important to every living organism on Earth

Ryan Woodard

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# Introduction

- Eddies get larger and move around.
- At a certain scale (the Rhine's length), the changing Coriolis parameter ends eddy growth.
- Energy goes into Rossby wave production.
- Rossby waves decay to smaller frequencies; this produces zonal flow.

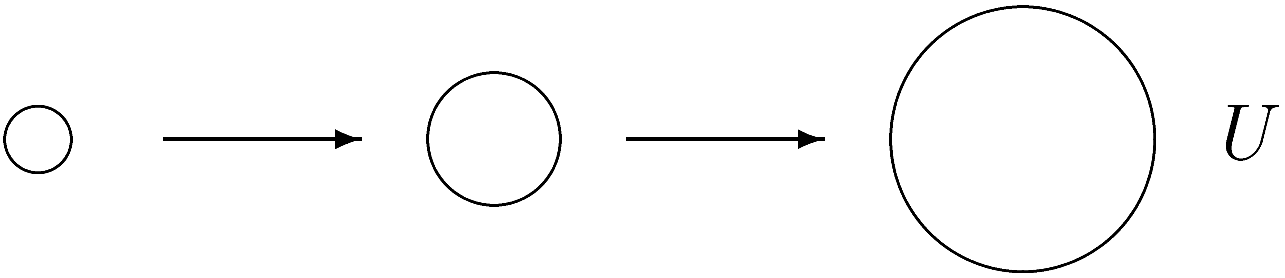
# Length scale at which Earth's rotation is important

| | | |
|---|---|---|
| time for one rotation of Earth | $\approx$ | time for fluid parcel to move $L$ at $U$ |

$$\frac{L}{U} \approx \frac{2\pi}{\Omega}$$

For $U \approx 1$ m/s , $L \approx 100$ km

In 2D turbulence, the inverse energy cascade produces larger eddies:




Relative vorticity of such an eddy is

$$\zeta = \nabla x \vec{u}$$

For eddies with a velocity $U$ at the perimeter,

$$\zeta = \frac{2U}{r}$$

# Potential vorticity must be conserved

$$q = \frac{\zeta + f}{h}$$

For constant $h$,

$$\zeta + f \approx \text{constant}$$

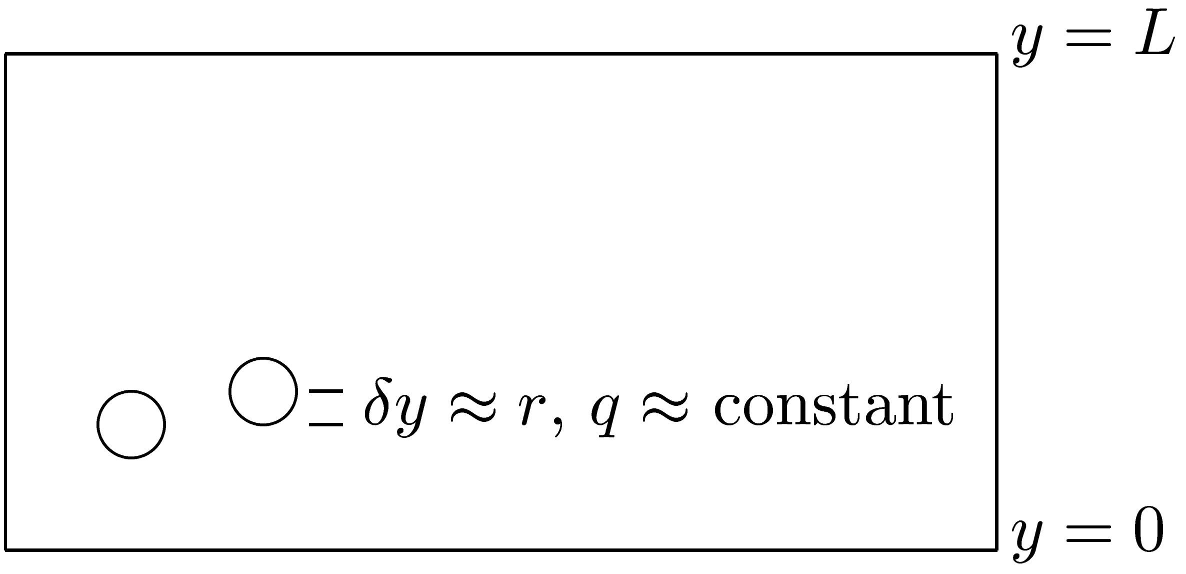
For an eddy

$$q = \frac{2U}{r} + f \approx \text{constant}$$

# On the $\beta$ plane, small eddies

$$q = \frac{2U}{r} + f_0 + \beta_0 r$$

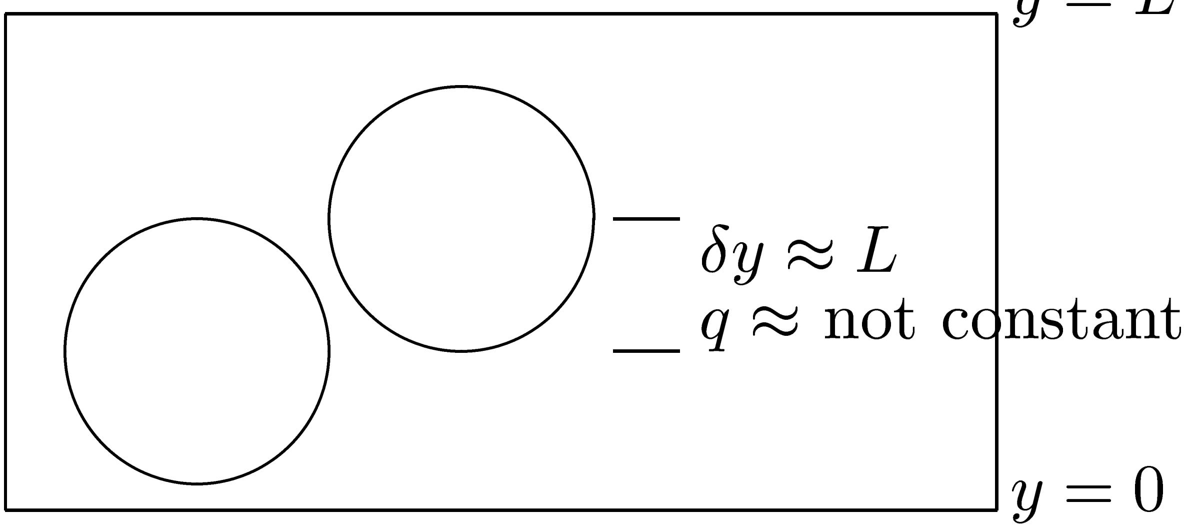
$$r \approx \delta y \ll L,\ \frac{2U}{r} \gg \beta_0 r$$



# On the $\beta$ plane, large eddies

$$q = \frac{2U}{r} + f_0 + \beta_0 r$$

$$r \approx \delta y \approx L,\ \frac{2U}{r} \approx \beta_0 r$$



# On the $\beta$ plane, large eddies

$$q = \frac{2U}{r} + f_0 + \beta_0 r$$

$$\frac{2U}{r} \approx \beta_0 r \Longrightarrow r = \sqrt{\frac{2U}{\beta_0}} \equiv L_R$$

$L_R$ is the Rhine's length

in ocean, $L_R \approx 100$ km

in atmos., $L_R \approx 1000$ km

## At $L_R$, eddies feel rotation effects

$$\frac{L_R}{U} \approx \tau \approx \frac{2\pi}{\Omega}$$

So flow takes so long to go around eddy, it begins to notice the Coriolis.

# What are the effects of rotation?

Remember our first, simplest solution on the $\beta$ plane:

## Rossby waves

Eddys at size limit ($L_R$) still contain energy.

That energy goes to drive Rossby waves.

# Rossby wave solutions

$$\eta \sim e^{i(lx+my-\omega t)}$$

$$u = u_0 + u_c e^{i(lx+my-\omega t)}$$

$$v = v_0 + v_c e^{i(lx+my-\omega t)}$$

Dispersion relation:

$$\omega \sim \frac{-\beta_0 l}{l^2+m^2}$$

# Rossby waves are unstable

They decay to Rossby waves with smaller frequencies.

$$\omega_1 \rightarrow \omega_2, \omega_3$$

$$\omega_2 < \omega_1 \text{ and } \omega_3 < \omega_1$$

# This decay brings about zonal flow

$\omega \to 0,\ l \to 0$

(remember: $\omega \sim \frac{-\beta_0 l}{l^2+m^2}$)

$u \to u_0 + e^{imy} \Longrightarrow u$ is independent of $x$, so is a constant zonal flow

# Conclusions

- Eddies get larger and move around.
- At Rhine's length, the changing Coriolis parameter ends eddy growth.
- Energy goes into Rossby wave production.
- Rossby waves decay to smaller frequencies which produces zonal flow.
- Could produce the zonal flows seen on Jupiter (see web project).
- Is why we don't have system size eddies on Earth.
- Allows trade winds and westerlies that were important for...

## . . . conclusions

Columbus to 'discover' this place, thereby setting into motion the genocide of native people, the slave trade, the destruction of healthy ecosystems, multitudes of extinctions and the inevitable control of the world by right-wing multinational conglomerate corporate puppet politicians and power-hungry CEOs.

That is why the Rhine's length is crucially important to every living organism on Earth.