

equations. We see that the pressure must necessarily be an intrinsic variable in fluid dynamical problems for there to be enough variables to satisfy the basic laws of mechanics.

For many particular problems it is convenient to use the equations referred to a coordinate system rather than the vectorial forms. Most often one uses Cartesian coordinates, but sometimes a curvilinear system is suggested by the geometry. Listed below are the forms taken by the continuity eqn (5.10) and the three components of the Navier-Stokes eqn (5.21) in Cartesian, cylindrical polar, and spherical polar co-ordinates:

1. Cartesian coordinates:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (5.23)$$

$$\rho \left[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right] = -\frac{\partial p}{\partial x} + \mu \left[ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right] + F_x \quad (5.24)$$

together with similar equations for  $v$  and  $w$ .

2. Cylindrical polar coordinates ( $r$  = distance from axis,  $\phi$  = azimuthal angle about axis,  $z$  = distance along axis):

$$\frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{1}{r} \frac{\partial u_\phi}{\partial \phi} + \frac{\partial u_z}{\partial z} = 0 \quad (5.25)$$

$$\rho \left[ \frac{\partial u_r}{\partial t} + u_r \frac{\partial u_r}{\partial r} + \frac{u_\phi}{r} \frac{\partial u_r}{\partial \phi} + u_z \frac{\partial u_r}{\partial z} - \frac{u_\phi^2}{r} \right] = -\frac{\partial p}{\partial r} + \mu \left[ \frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r} \frac{\partial u_r}{\partial r} - \frac{u_r}{r^2} + \frac{1}{r^2} \frac{\partial^2 u_r}{\partial \phi^2} + \frac{\partial^2 u_r}{\partial z^2} - \frac{2}{r^2} \frac{\partial u_\phi}{\partial \phi} \right] + F_r \quad (5.26)$$

$$\rho \left[ \frac{\partial u_\phi}{\partial t} + u_r \frac{\partial u_\phi}{\partial r} + \frac{u_r u_\phi}{r} + \frac{u_\phi}{r} \frac{\partial u_\phi}{\partial \phi} + u_z \frac{\partial u_\phi}{\partial z} \right] = -\frac{1}{r} \frac{\partial p}{\partial \phi} + \mu \left[ \frac{\partial^2 u_\phi}{\partial r^2} + \frac{1}{r} \frac{\partial u_\phi}{\partial r} - \frac{u_\phi}{r^2} + \frac{1}{r^2} \frac{\partial^2 u_\phi}{\partial \phi^2} + \frac{\partial^2 u_\phi}{\partial z^2} + \frac{2}{r^2} \frac{\partial u_r}{\partial \phi} \right] + F_\phi \quad (5.27)$$

$$\rho \left[ \frac{\partial u_z}{\partial t} + u_r \frac{\partial u_z}{\partial r} + \frac{u_\phi}{r} \frac{\partial u_z}{\partial \phi} + u_z \frac{\partial u_z}{\partial z} \right] = -\frac{\partial p}{\partial z} + \mu \left[ \frac{\partial^2 u_z}{\partial r^2} + \frac{1}{r} \frac{\partial u_z}{\partial r} + \frac{1}{r^2} \frac{\partial^2 u_z}{\partial \phi^2} + \frac{\partial^2 u_z}{\partial z^2} \right] + F_z \quad (5.28)$$

3. Spherical polar coordinates ( $r$  = distance from origin,  $\theta$  = angular displacement from reference direction,  $\phi$  = azimuthal angle about line