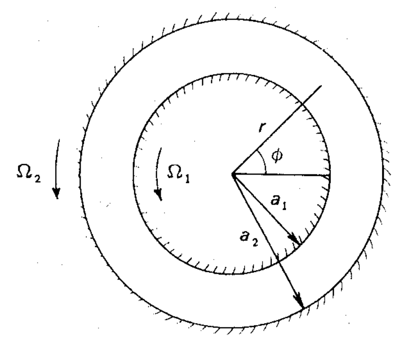

FIG. 9.1 Definition sketch for rotating Couette flow (z-axis is normal to paper).

assumptions, and the azimuthal and radial components of the Navier–Stokes equation become

$$0 = \mu\left(\frac{d^2u_\phi}{dr^2} + \frac{1}{r}\frac{du_\phi}{dr} - \frac{u_\phi}{r^2}\right) \tag{9.3}$$

$$-\frac{\rho u_\phi^2}{r} = -\frac{dp}{dr} \tag{9.4}$$

with the boundary conditions

$$u_\phi = \Omega_1 a_1 \quad \text{at} \quad r = a_1; \quad u_\phi = \Omega_2 a_2 \quad \text{at} \quad r = a_2. \tag{9.5}$$

The first equation can be solved to give u_ϕ and this is then put into the second equation to give p. Thus, the distribution of the azimuthal velocity across the annulus is determined by the balance of viscous stresses, whilst the pressure distribution is determined by the balance between a radial pressure gradient and the centrifugal force associated with the circular motion.

The solution for u_ϕ (obtained by working in terms of the variable u_ϕ/r) is

$$u_\phi = Ar + B/r \tag{9.6}$$

where

$$A = (\Omega_2 a_2^2 - \Omega_1 a_1^2)/(a_2^2 - a_1^2), \quad B = (\Omega_1 - \Omega_2)a_1^2 a_2^2/(a_2^2 - a_1^2). \tag{9.7}$$

The torque Σ_1 acting on the inner cylinder (per unit length in the z-direction) is given by the viscous stress† $\mu[r\partial(u_\phi/r)/\partial r]_{r=a_1}$ multiplied by the area $2\pi a_1$ and by the radius a_1; i.e.

$$\Sigma_1 = 4\pi\mu a_1^2 a_2^2(\Omega_2 - \Omega_1)/(a_2^2 - a_1^2). \tag{9.8}$$

† That transformation to polar coordinates gives an expression of this form is to be expected from the fact that there will be no stress in rigid-body rotation, $u_\phi \propto r$.