t principles, starting with the fact that
; with the flow is conserved; that is,

criterion for compressibility effects to
ow.
accelerated to a speed of order U in a
r the motion of fluid surrounding the
en Ψ is small compared with L/U (so
d first the criterion for the fractional
sider whether this is sufficient for the

$$+\rho\nabla\cdot\boldsymbol{u}=0$$

pressible flow.)

s of Fig. 2.1 is suddenly turned off.
attern throughout the pipe is that of
e time in which the height of the water
s original value.
f the inertial effects associated with (a)
e, and (b) the changing velocity within

diameter 1 mm and length 10 m leading
n $10^{-2}\,\text{m}^2$. Check whether the above
100 mm.

tion

$$\Omega x \quad w = w_0$$

$$\sin(\Omega z/w_0+\alpha)$$

by substitution of the result.)

w, a flow is described in a Lagrangian
the coordinates (x, y, z) of the fluid
each flow, (a) formulate the equations
he Eulerian equations ($u = u(x, y, z, t)$
steady, (d) consider whether the flow
equation, and (e) derive a general
ure of the flows?

$$z = z_0\exp(t/s)$$

$$z = z_0\exp(2t/s)(1+t/s)^{-2}$$

nstant > 0).

11. Derive the general equation for a streamline in the two-dimensional flow (an approximate form of Rossby wave)

$$u = u_0, \quad v = v_0\cos(kx-\alpha t)$$

where u_0, v_0, k, and α are constants. At $t=0$, what is the equation for the streamline passing through $x=0$, $y=0$?

Derive also the equation for the path of the particle which is at $x=0$, $y=0$ at time $t=0$.

Comment briefly on the comparison of the streamline and the particle path in the two limiting cases, $\alpha=0$ and $k=0$.

12. Show geometrically that for cylindrical polar coordinates

$$\partial\hat{\boldsymbol{r}}/\partial\phi=\hat{\boldsymbol{\phi}} \quad \text{and} \quad \partial\hat{\boldsymbol{\phi}}/\partial\phi=-\hat{\boldsymbol{r}}.$$

Hence, by writing

$$\boldsymbol{u}=u_r\hat{\boldsymbol{r}}+u_\phi\hat{\boldsymbol{\phi}}+u_z\hat{\boldsymbol{z}}, \quad \nabla=\hat{\boldsymbol{r}}\frac{\partial}{\partial r}+\frac{\hat{\boldsymbol{\phi}}}{r}\frac{\partial}{\partial\phi}+\hat{\boldsymbol{z}}\frac{\partial}{\partial z}$$

show (1) that the inertia term takes the form given in eqns (5.26), (5.27), and (5.28); and (2) that, for the case when $u_r=u_z=0$ and u_ϕ depends on r alone, the viscous term takes the form given in eqn (9.3).

Note: the order of operations is indicated by writing $\boldsymbol{u}\cdot\nabla\boldsymbol{u}=(\boldsymbol{u}\cdot\nabla)\boldsymbol{u}$ and $\nabla^2\boldsymbol{u}=(\nabla\cdot\nabla)\boldsymbol{u}$.

13. (1) Show that, for an unsteady flow in which only the x-component of velocity is non-zero and this varies only in the y-direction, the Navier–Stokes equation reduces to a form analogous to the equation of unsteady one-dimensional heat conduction in a solid.

(2) An effectively infinite flat plate bounding a semi-infinite expanse of fluid oscillates in its own plane with velocity

$$U=U_0\sin\omega t.$$

Supposing that the induced fluid motion is an oscillation of the same frequency, how do the amplitude and phase vary with distance from the plate?

(3) For a geometry similar to that in (2), the plate is suddenly brought into motion at time $t=0$ and then moves in its own plane with constant velocity U_0. Both the plate and the fluid were at rest for $t<0$. Show that, for $t>0$, the fluid velocity is

$$u=U_0\,\mathrm{erfc}[y/2(\nu t)^{1/2}].$$

Find the force per unit area (as a function of time) needed to produce this motion. Hence, find the total work done after any given time, and determine what proportion of this work has appeared as kinetic energy and what proportion has been dissipated.

$$\left[\mathrm{erfc}\,x=\frac{2}{\sqrt{\pi}}\int_x^\infty\exp(-x^2)\,\mathrm{d}x;\quad \mathrm{erfc}\,0=1;\quad \int_0^\infty(\mathrm{erfc}\,x)^2\,\mathrm{d}x=\frac{2-\sqrt{2}}{\sqrt{\pi}}\right].$$