is called the *potential vorticity*. The preceding analysis interprets potential vorticity as circulation per volume. This quantity, as will be shown on a numerous occasions in this book, plays a fundamental role in geophysical flows. Note that equation (4-26) could have been derived directly from (4-20) and (4-22) without recourse to the introduction of the variable ds.

Let us now go full circle and return to rapidly rotating flows, those in which the Coriolis force dominates. In this case, the Rossby number is much less than unity ($Ro = U/\Omega L \ll 1$), which implies that the relative vorticity ($\zeta = \partial v/\partial x - \partial u/\partial y$, scaling like U/L) is negligible in front of the ambient vorticity (f, scaling like Ω). The potential vorticity reduces to

$$q = \frac{f}{h} \qquad (4\text{-}28)$$

which, if f is constant—such as in a rotating laboratory tank or for geophysical patterns of modest meridonal extent—implies that each fluid column must conserve its height h. In particular, if the upper boundary is horizontal, fluid parcels must follow isobaths.

PROBLEMS

4-1. A laboratory experiment is conducted in a cylindrical tank 20 cm in diameter, filled with homogeneous (15 cm deep at the center) water and rotating at 30 rpm. A steady flow field with maximum velocities of 1 cm/s is generated by a source-sink device. The water viscosity is 10^{-6} m^2/s. Verify that this flow field meets the conditions of geostrophy.

4-2. (Generalization of the Taylor–Proudman theorem) By reinstating the f_*-terms of equations (3-13), (3-14), and (3-17) into (4-2) through (4-4), show that motions in fluids rotating rapidly around an axis not parallel to gravity exhibit columnar behavior in the direction of the axis of rotation.

4-3. Demonstrate the assertion made at the end of Section 4-2, namely, that geostrophic flows between irregular bottom and top boundaries are constrained to be directed along lines of constant fluid depth.

4-4. Establish equation (4-23) for the evolution of a parcel's horizontal cross-section from first principles.

4-5. In a fluid of depth H rapidly rotating at the rate Ω (Figure 4-7), there exists a uniform flow U. Along the bottom (fixed), there is an obstacle of height H' ($< H/2$), around which the flow is locally deflected, leaving a quiescent Taylor column. A rigid lid, translating in the direction of the flow at the speed $2U$, has a protrusion identical to the bottom obstacle, also locally deflecting the otherwise uniform flow and entraining another quiescent Taylor column. The two obstacles are aligned with the motion axis so that there will be a time when both are superimposed. Assuming that the fluid is homogeneous and frictionless, what do you think will happen to the Taylor columns?

4-6. As depicted in Figure 4-8, a vertically uniform but laterally sheared coastal current must climb a bottom escarpment. Assuming that the jet velocity still vanishes offshore, determine