Physics 211

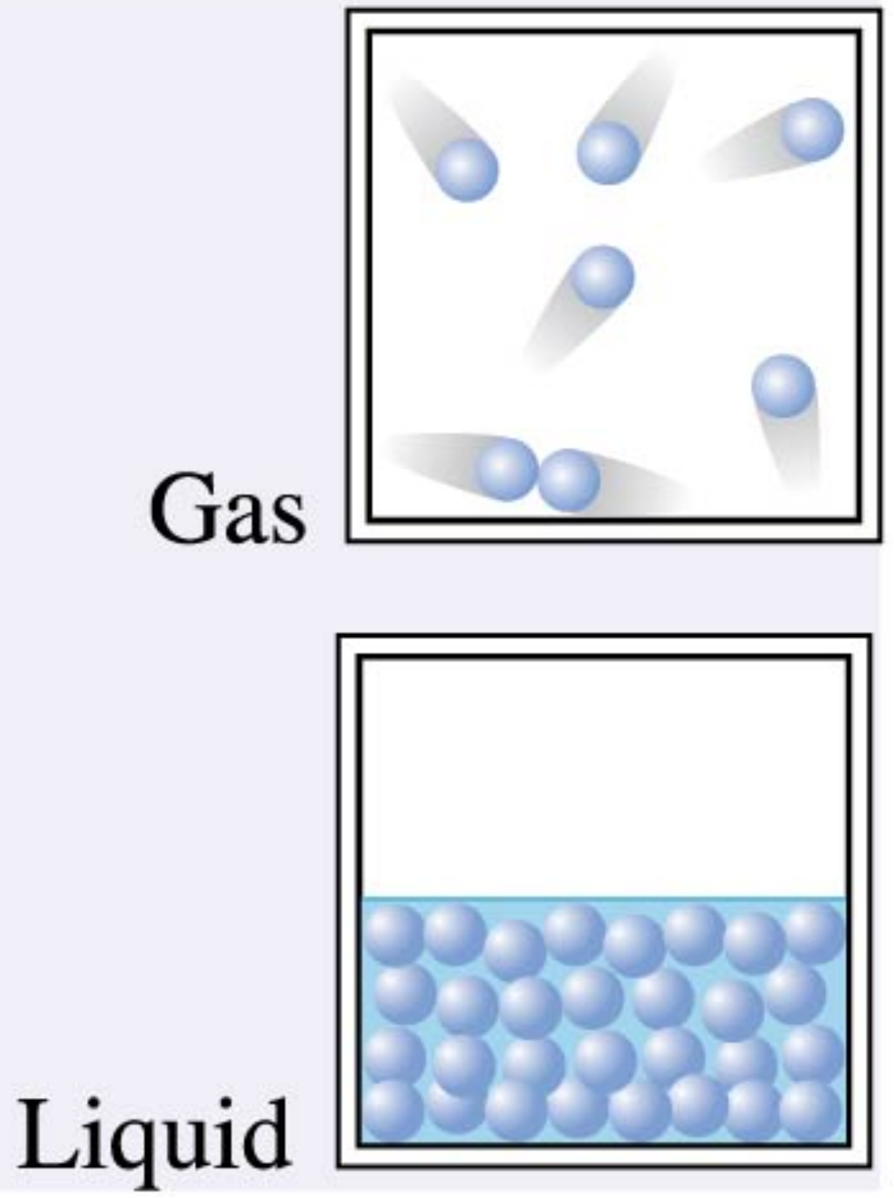
Lecture 40

David Newman

What is a fluid?

A fluid is a substance that flows. Both gases and liquids are fluids.

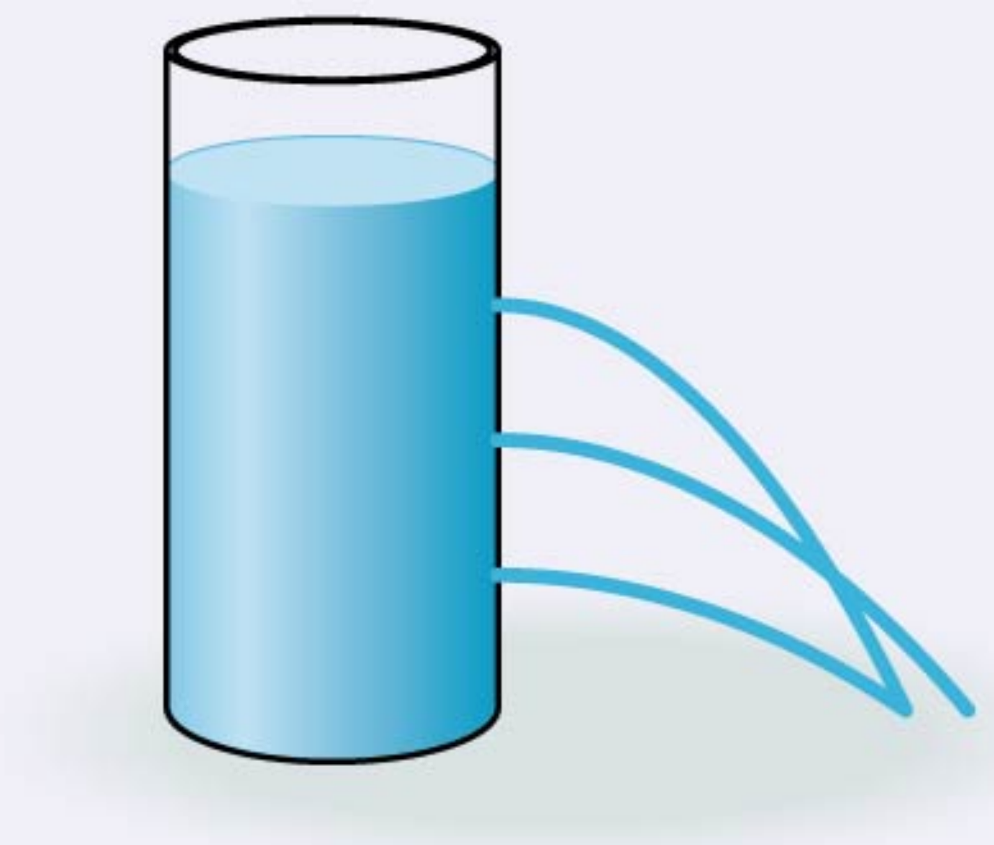
- A gas is compressible. The molecules move freely with few interactions.
- A liquid is incompressible. The molecules are weakly bound to one another.



What is pressure?

Fluids exert forces on the walls of their containers. Pressure is the force-to-area ratio F/A.

- Pressure in liquids, called hydrostatic pressure, is due to gravity. Pressure increases with depth.
- Pressure in gases is primarily thermal. Pressure is constant in a container.

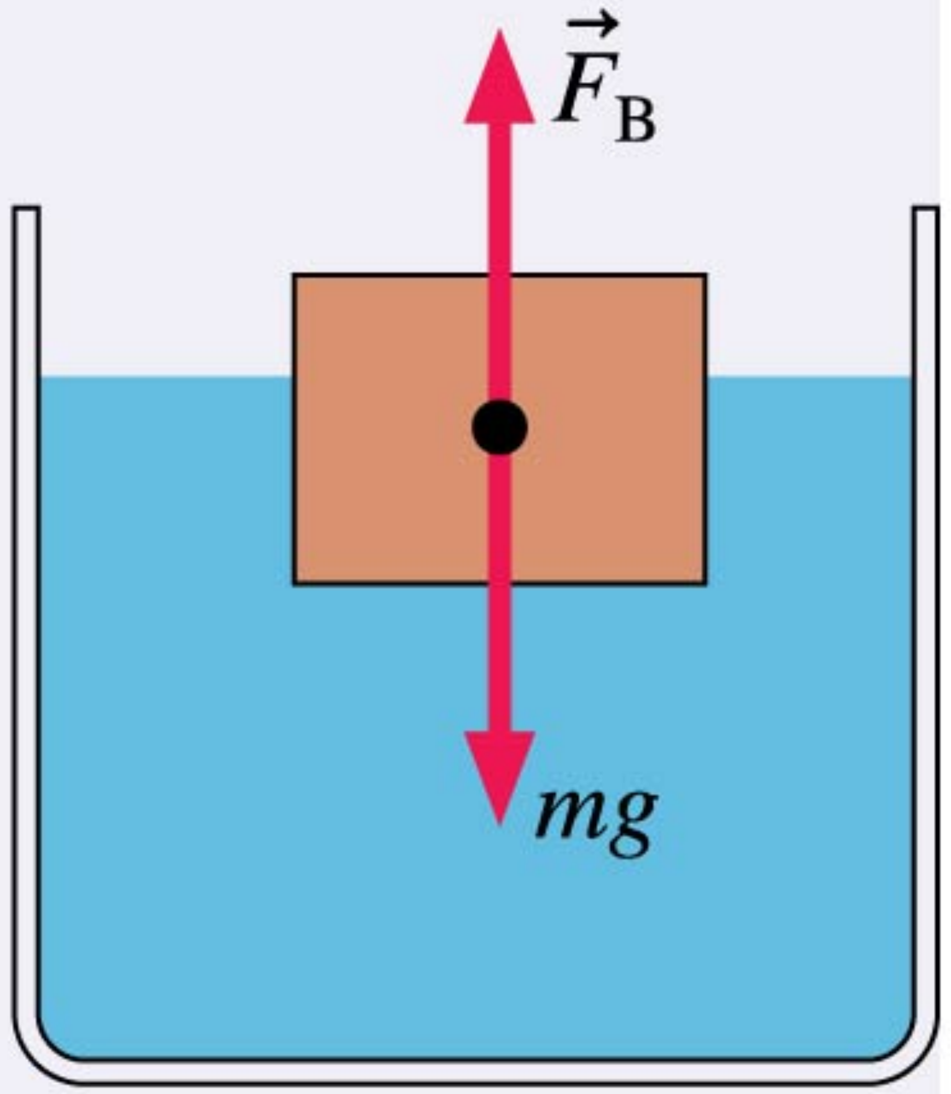


$P = \frac{F}{A}$

What is buoyancy?

Buoyancy is the upward force a fluid exerts on an object.

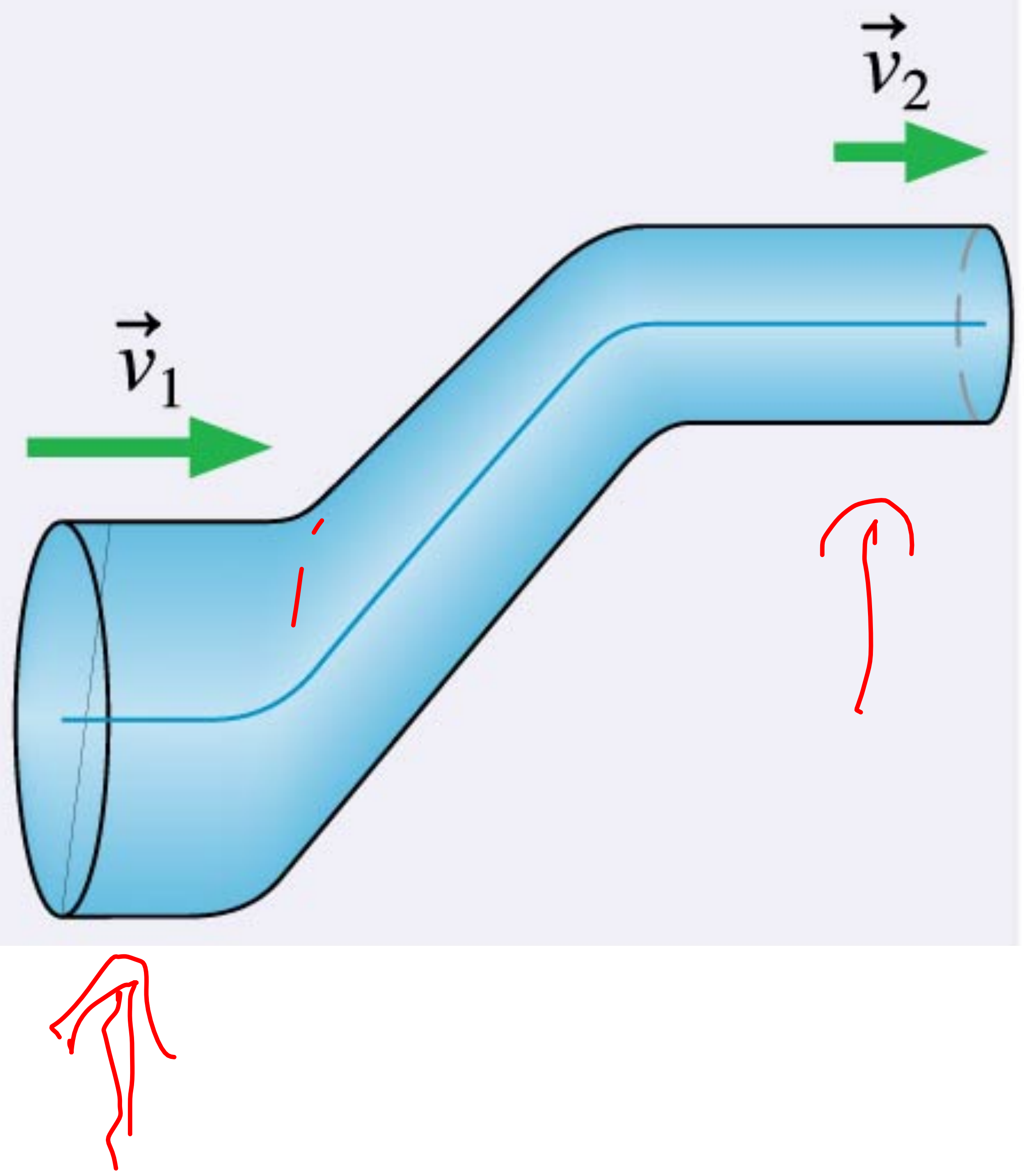
- Archimedes' principle says that the buoyant force equals the weight of the displaced fluid.
- An object floats if the buoyant force is sufficient to balance the object's weight.



« LOOKING BACK Section 6.1 Equilibrium

How does a fluid flow?

An ideal fluid—an incompressible, nonviscous fluid flowing smoothly—flows along streamlines. Bernoulli's equation, a statement of energy conservation, relates the pressures, speeds, and heights at two points on a streamline.

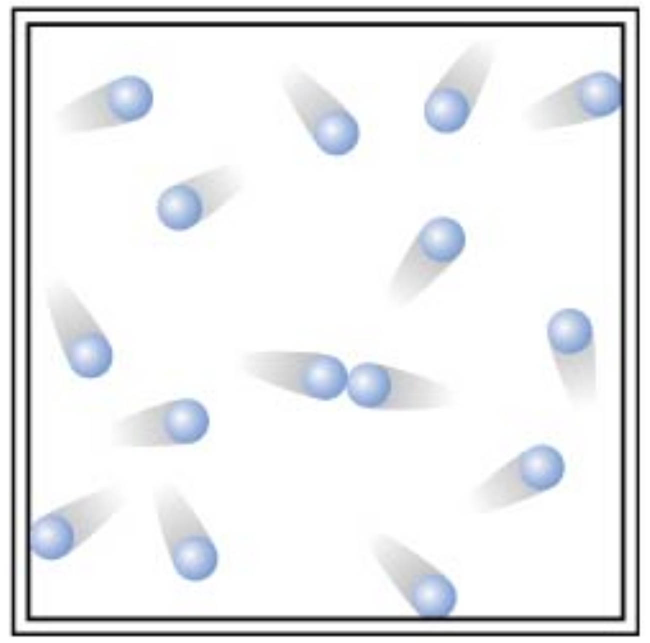


MODEL 14.1

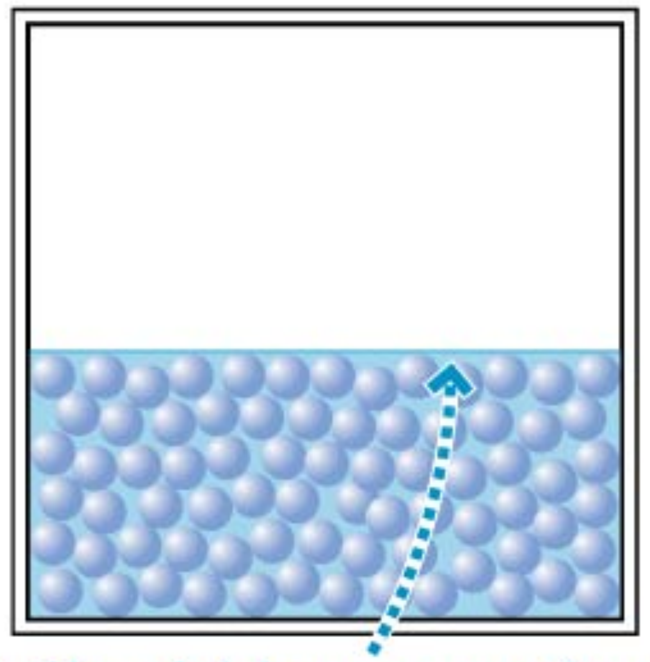
Molecular model of gases and liquids

Gases and liquids are fluids—they flow and exert pressure.

- **Gases**
  - Molecules move freely through space.
  - Molecules do not interact except for occasional collisions with each other or the walls.
  - Molecules are far apart, so a gas is *compressible*.
- **Liquids**
  - Molecules are weakly bound and stay close together.
  - A liquid is *incompressible* because the molecules can't get any closer.
  - Weak bonds allow the molecules to move around.



A gas fills the container.



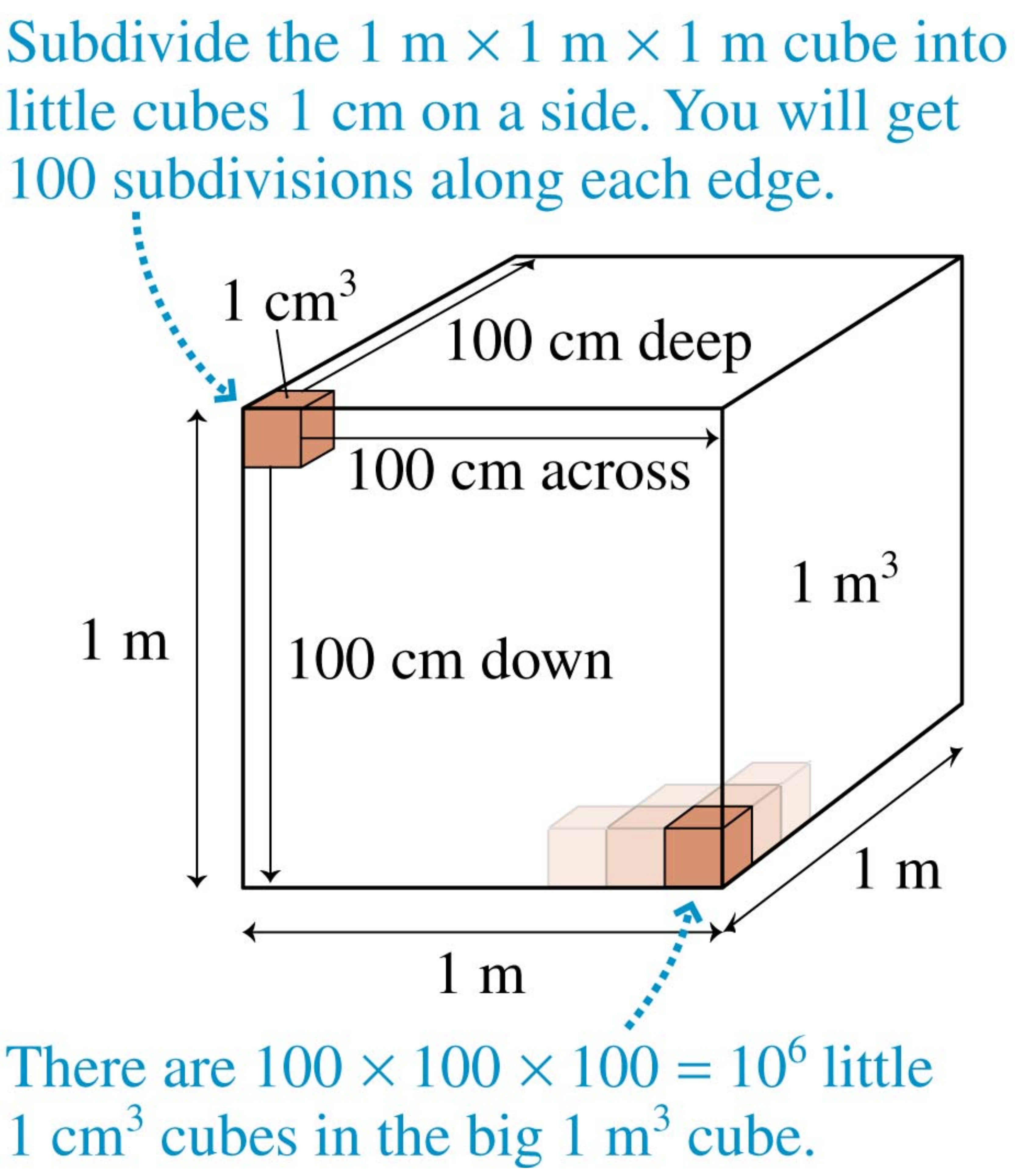
A liquid has a surface.

- An important parameter of a macroscopic system is its volume V.
- The S.I. unit of volume is m^3.
- Some unit conversions:

$$1\ m^3 = 1000\ L$$

$$1\ L = 1000\ cm^3$$

$$1\ m^3 = 10^6\ cm^3$$



Density

- The ratio of an object's or material's mass to its volume is called the **mass density**, or sometimes simply "the density."

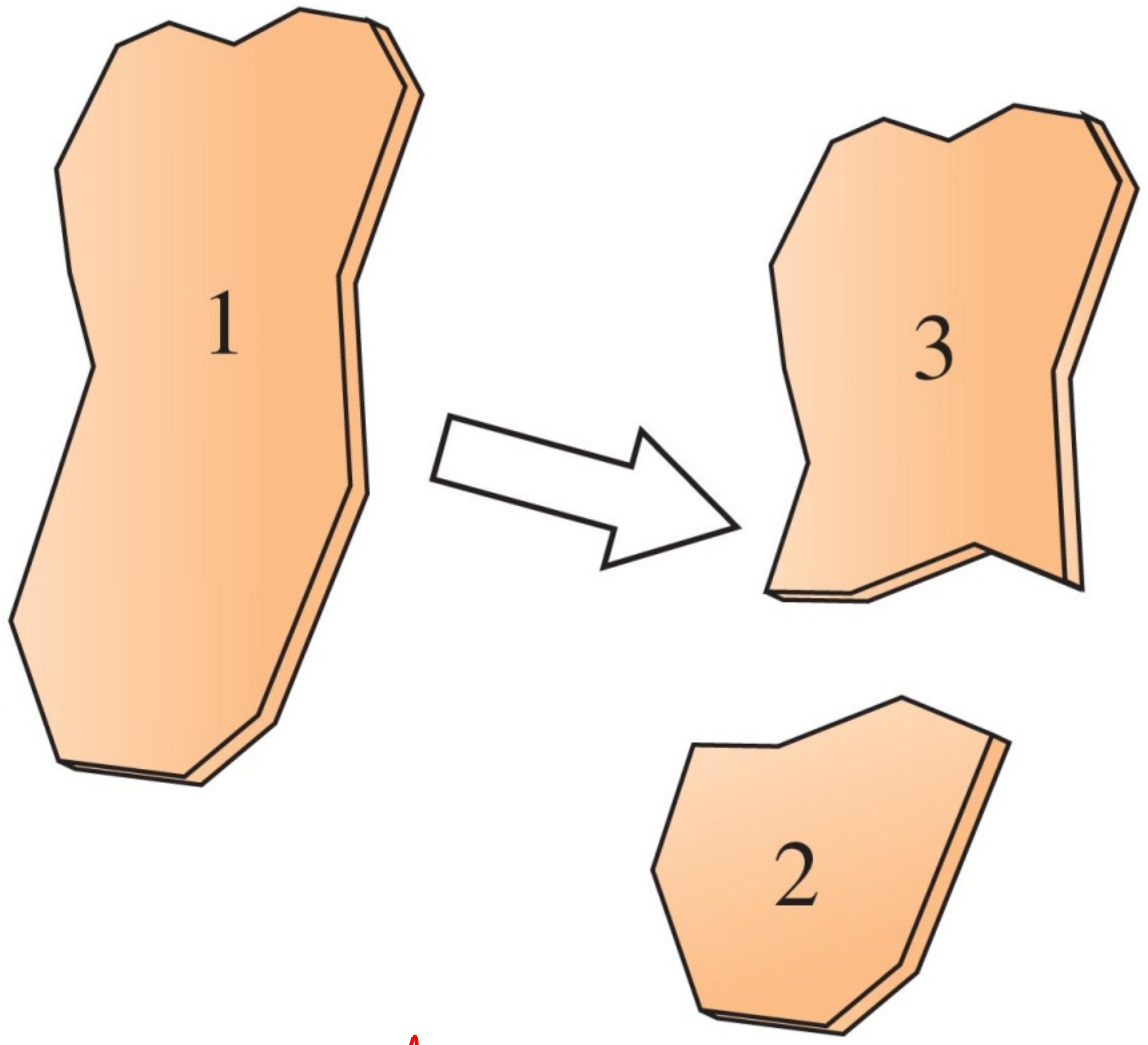
$$\rho = \frac{m}{V} \quad \text{(mass density)}$$

- The SI units of mass density are $\mathrm{kg/m^3}$.

QuickCheck 14.1

A piece of glass is broken into two pieces of different size. How do their densities compare?

A. $\rho_1 > \rho_3 > \rho_2$
B. $\rho_1 = \rho_3 = \rho_2$
C. $\rho_1 < \rho_3 < \rho_2$



same material

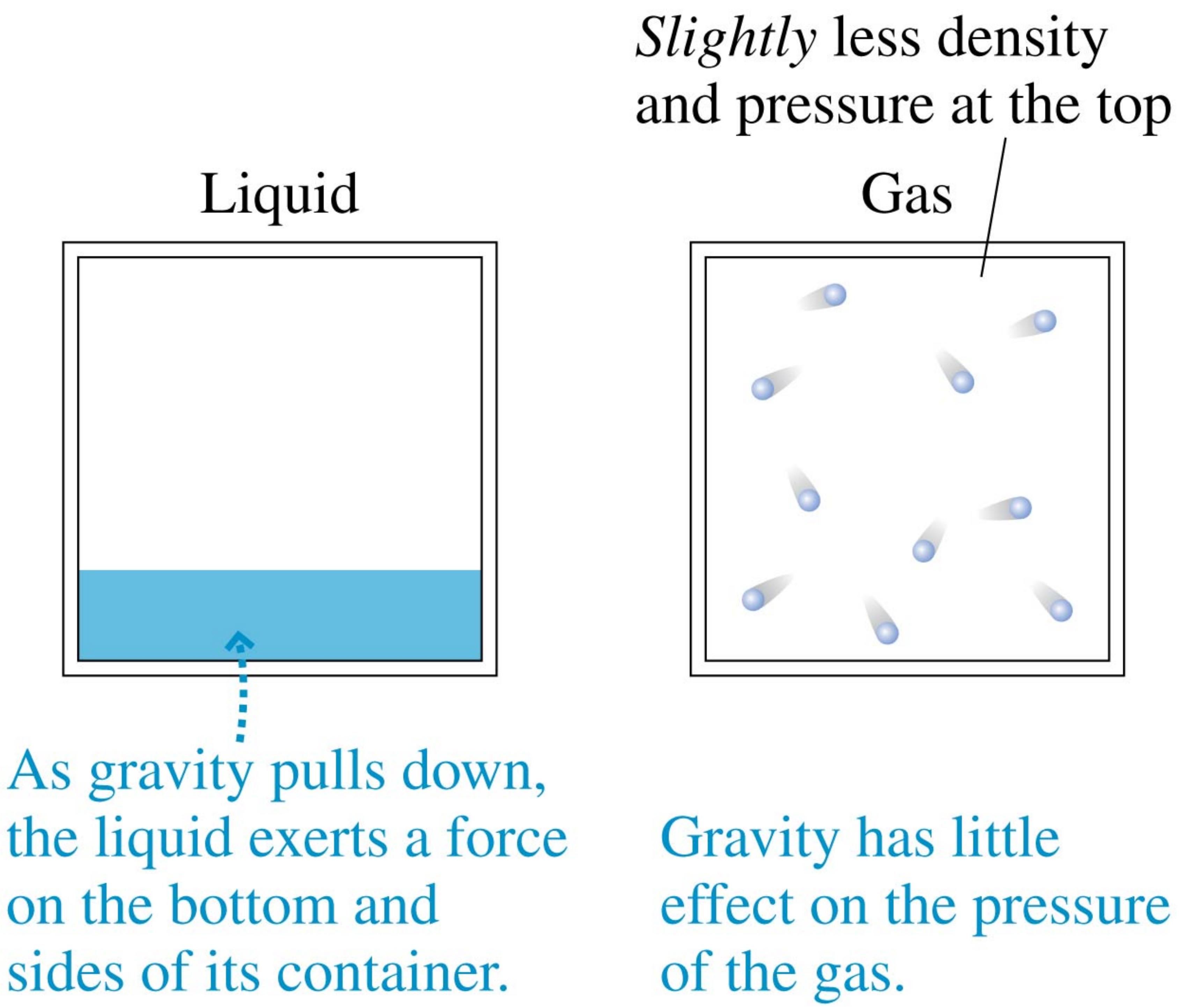
Densities of Various Fluids

TABLE 14.1 Densities of fluids at standard temperature (0°C) and pressure (1 atm)

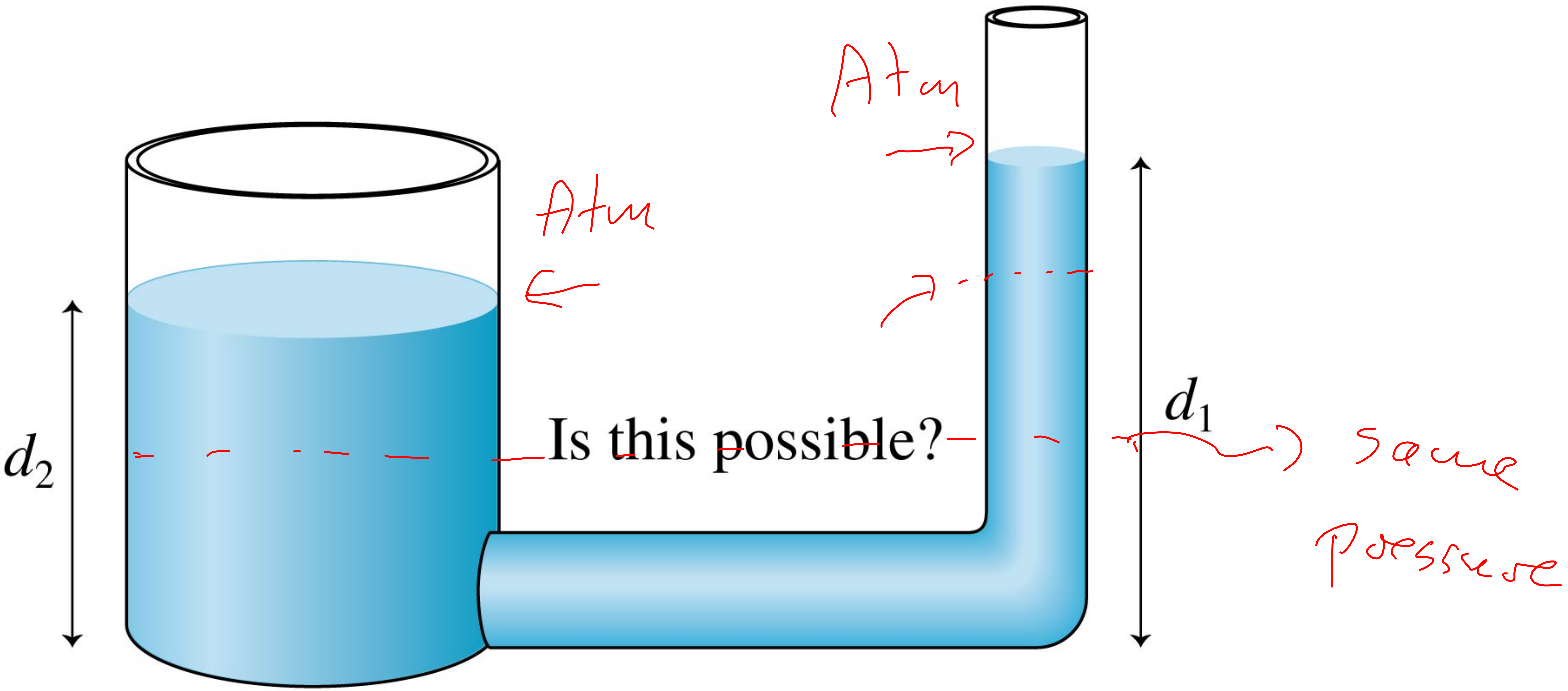
Substance	ρ (kg/m^3)
Helium gas	0.18
Air	1.29
Gasoline	680
Ethyl alcohol	790
Benzene	880
Oil (typical)	900
Water	1000
Seawater	1030
Glycerin	1260
Mercury	13,600

Pressure

- There are two contributions to the pressure in a container of fluid:
  1. A *gravitational contribution*, due to gravity pulling down on the liquid or gas.
  2. A *thermal contribution*, due to the collisions of freely moving gas molecules within the walls, which depends on gas temperature.



Liquids in Hydrostatic Equilibrium

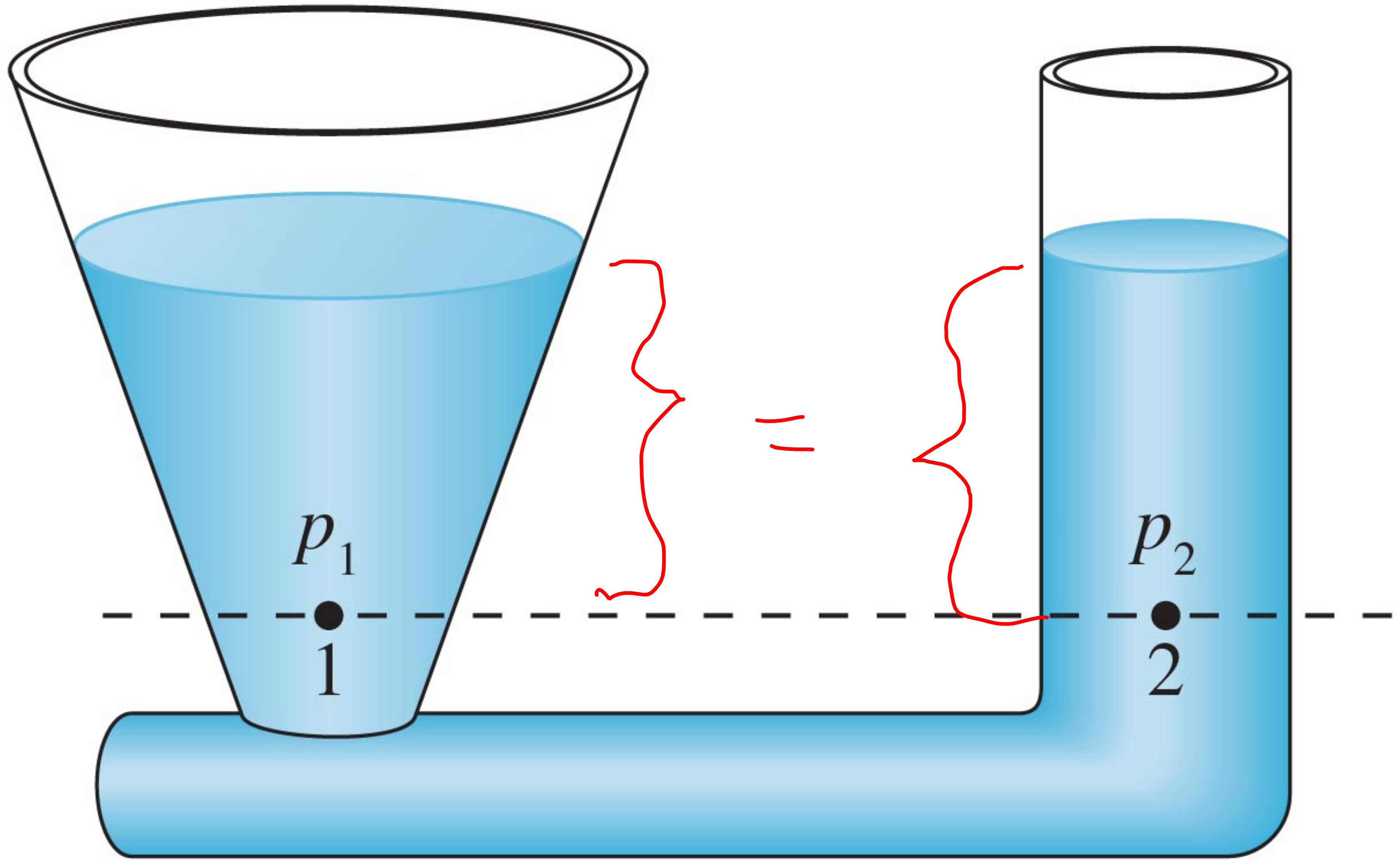


- No!
- A connected liquid in hydrostatic equilibrium rises to the same height in all open regions of the container.

QuickCheck 14.2

What can you say about the pressures at points 1 and 2?

A. $p_1 > p_2$
B. $p_1 = p_2$
C. $p_1 < p_2$

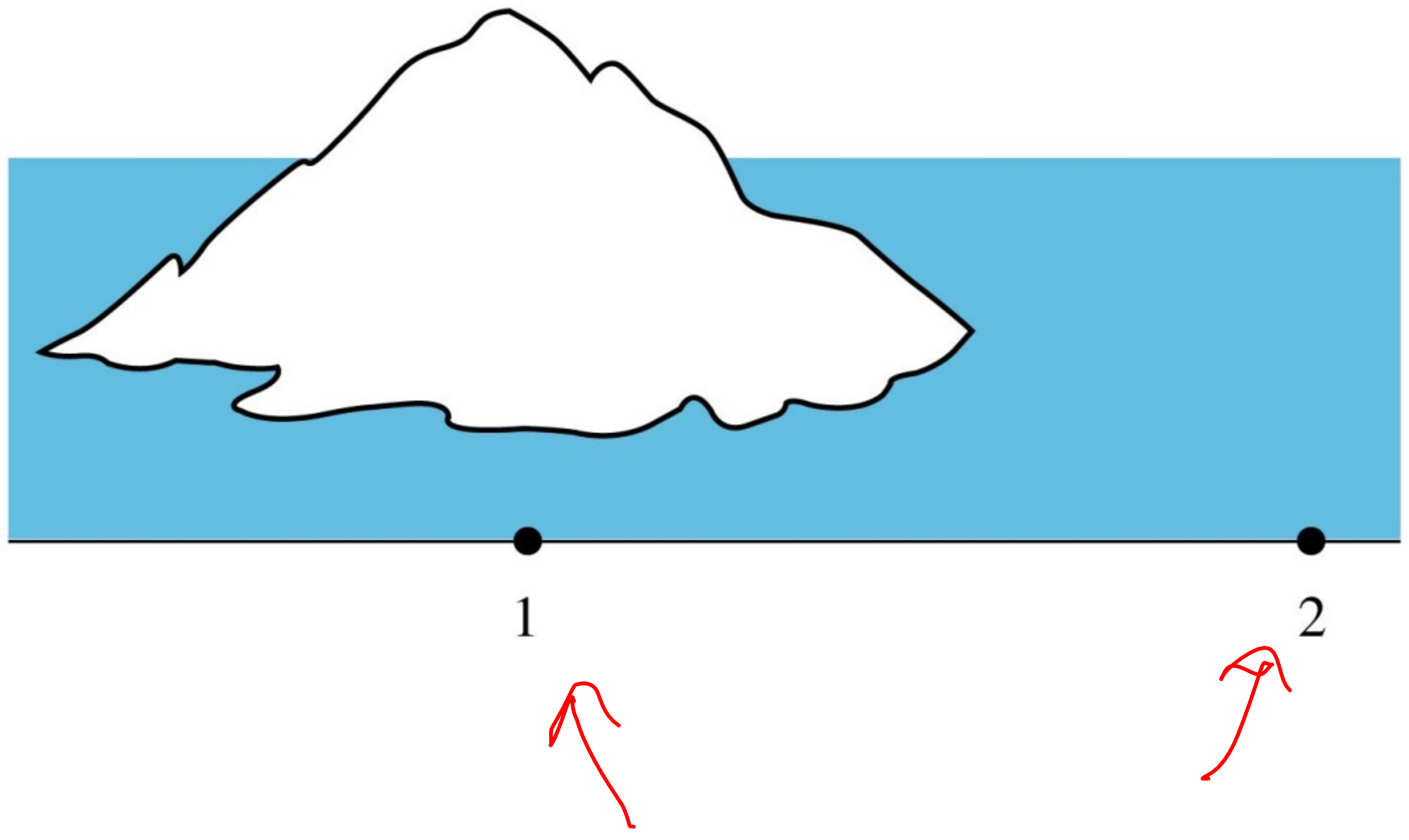


$P = \frac{F}{A}$ is the same

QuickCheck 14.3

An iceberg floats in a shallow sea. What can you say about the pressures at points 1 and 2?

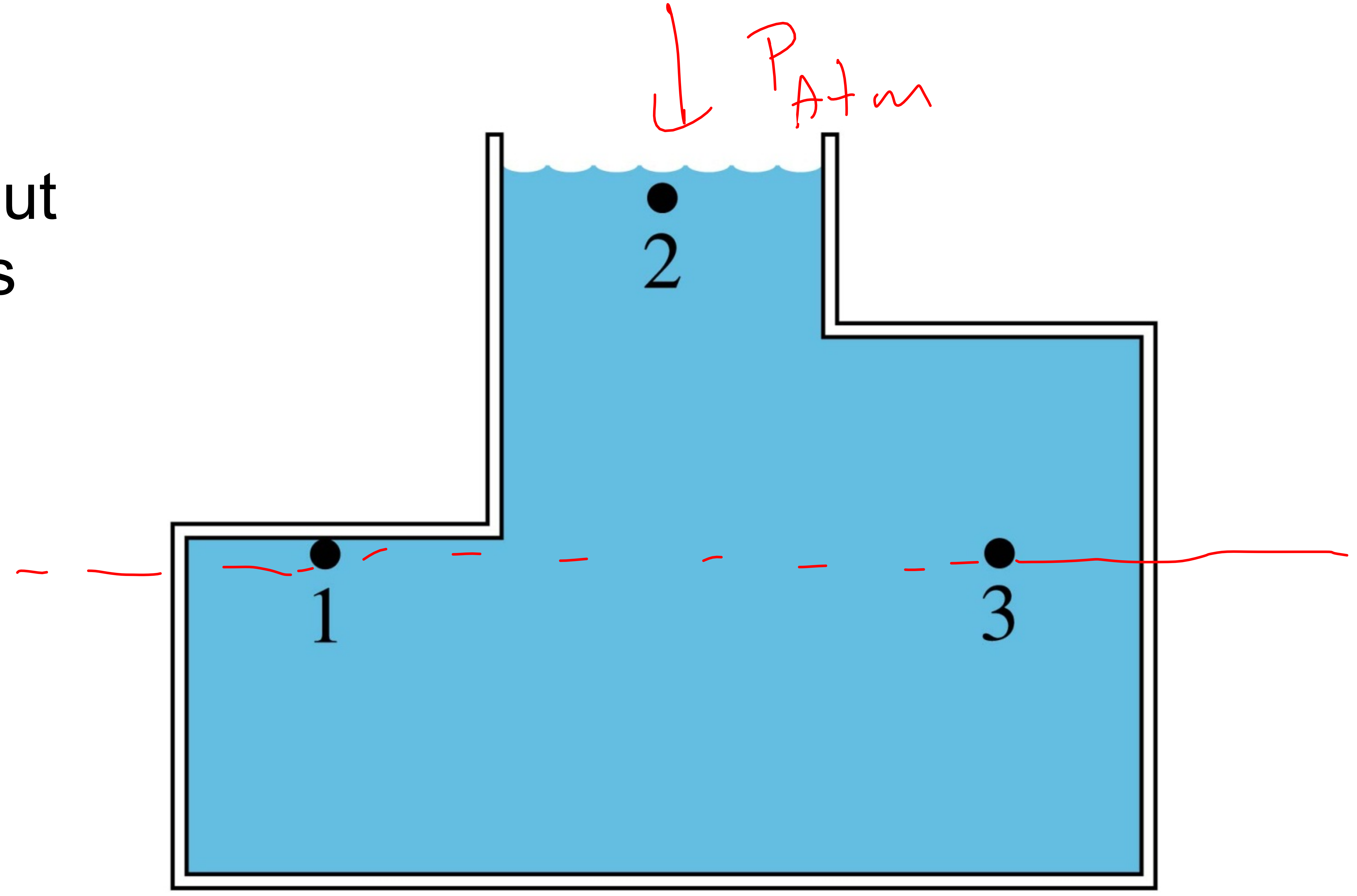
A. $p_1 > p_2$
B. $p_1 = p_2$
C. $p_1 < p_2$



QuickCheck 14.4

What can you say about the pressures at points 1, 2, and 3?

A. $p_1 = p_2 = p_3$
B. $p_1 = p_2 > p_3$
C. $p_3 > p_1 = p_2$
D. $p_3 > p_1 > p_2$
E. $p_1 = p_3 > p_2$



P = ρgh → depth

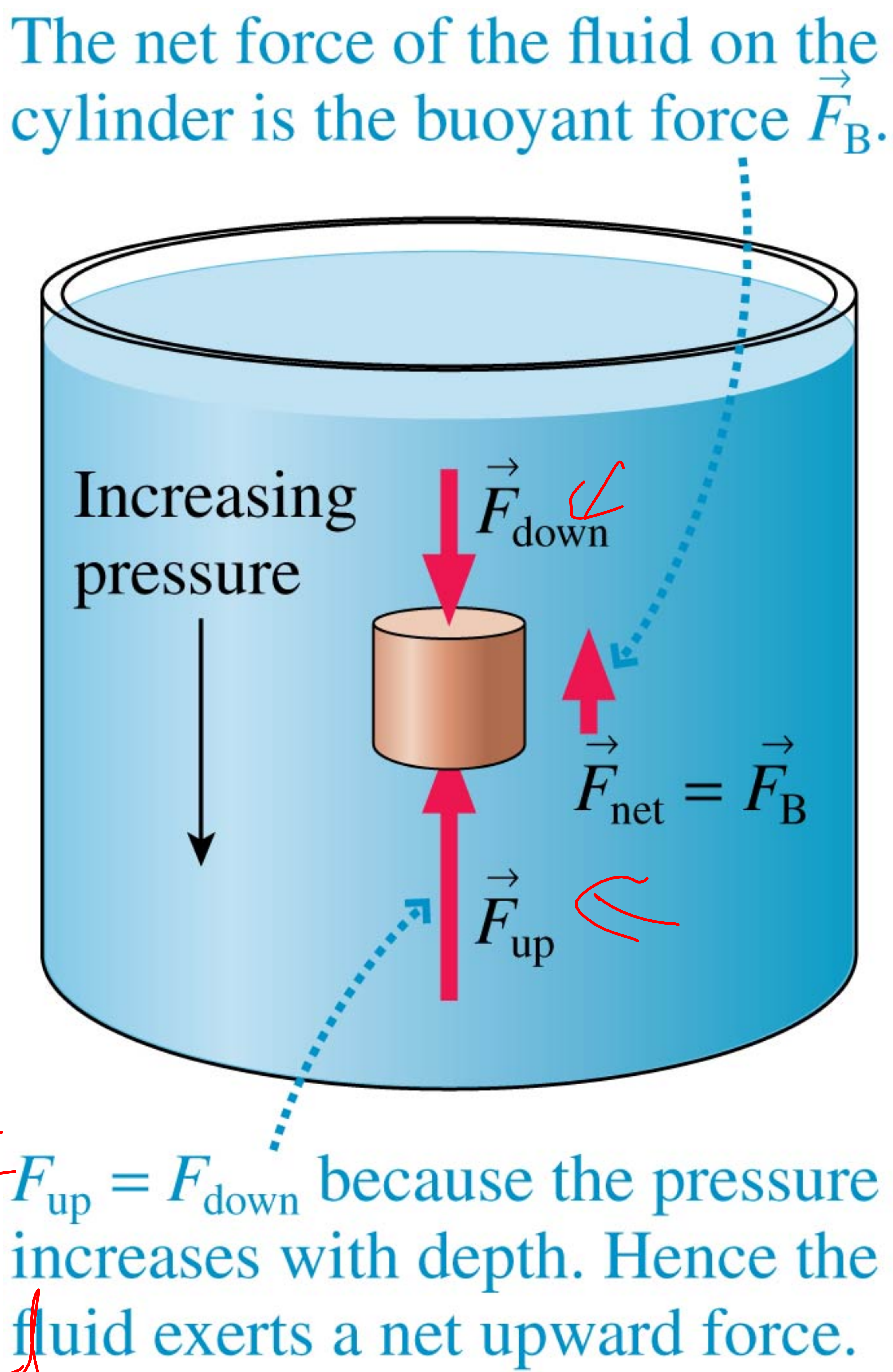
P tot = P Atm + ρgh

Buoyancy

- Consider a cylinder submerged in a liquid.
- The pressure in the liquid increases with depth.
- Both cylinder ends have equal area, so $F_{up} > F_{down}$
- The pressure in the liquid exerts a *net upward force* on the cylinder:

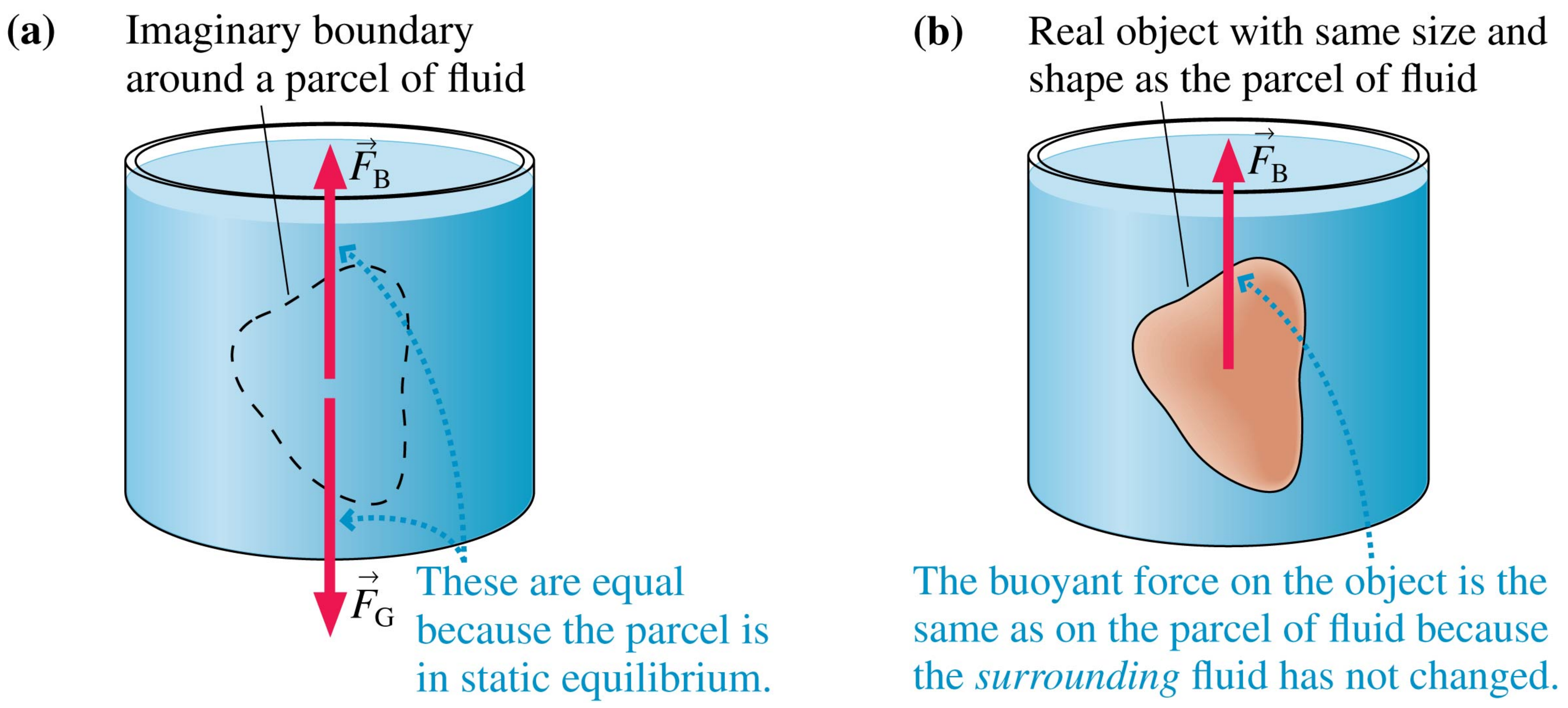
$$F_{net} = F_{up} - F_{down}$$

- This is the buoyant force.



buoyant force is the weight of the fluid displaced

Buoyancy



- The buoyant force on an object is the same as the buoyant force on the fluid it displaces.

Buoyancy

- When an object (or portion of an object) is immersed in a fluid, it displaces fluid.
- The **displaced fluid's** volume equals the volume of the portion of the object that is immersed in the fluid.

Archimedes' principle A fluid exerts an upward buoyant force $\vec{F}_B$ on an object immersed in or floating on the fluid. The magnitude of the buoyant force equals the weight of the fluid displaced by the object.

- Suppose the fluid has density ρ_f and the object displaces volume V_f of fluid.
- Archimedes' principle in equation form is

$$F_B = \rho_f V_f g$$

QuickCheck 14.5

A heavy lead block and a light aluminum block of equal sizes are both submerged in water. Upon which is the buoyant force greater?

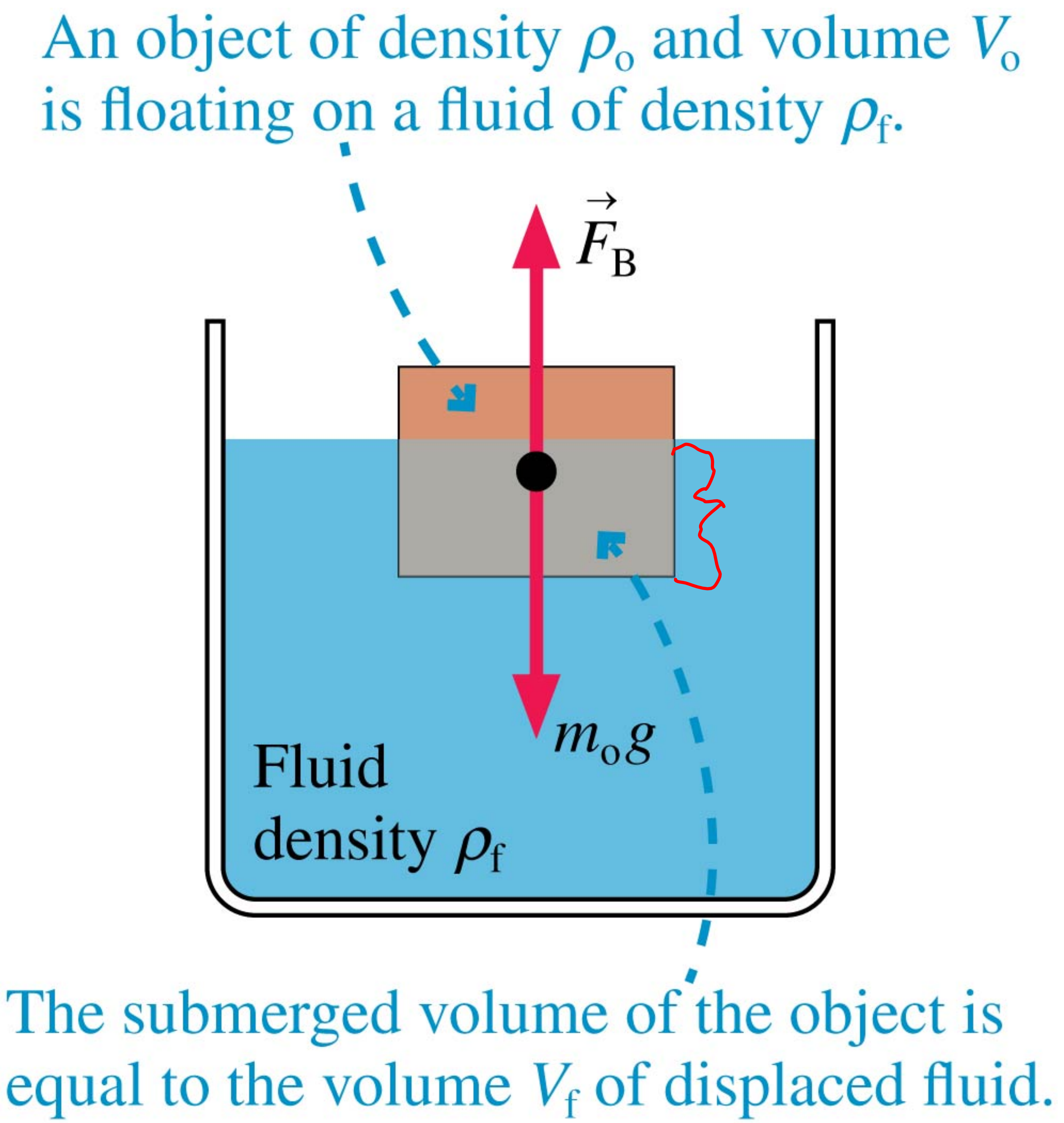
A. On the lead block
B. On the aluminum block
C. They both experience the same buoyant force.

- The volume of fluid displaced by a floating object of uniform density is:

$$F_B = \rho_f V_f g = m_o g = \rho_o V_o g$$

- The volume of the displaced fluid is *less than* the volume of the uniform-density object:

$$V_f = \frac{\rho_o}{\rho_f} V_o < V_o$$



A Floating Object

- Most icebergs break off glaciers and are fresh-water ice with a density of 917 kg/m³.
- The density of seawater is 1030 kg/m³:

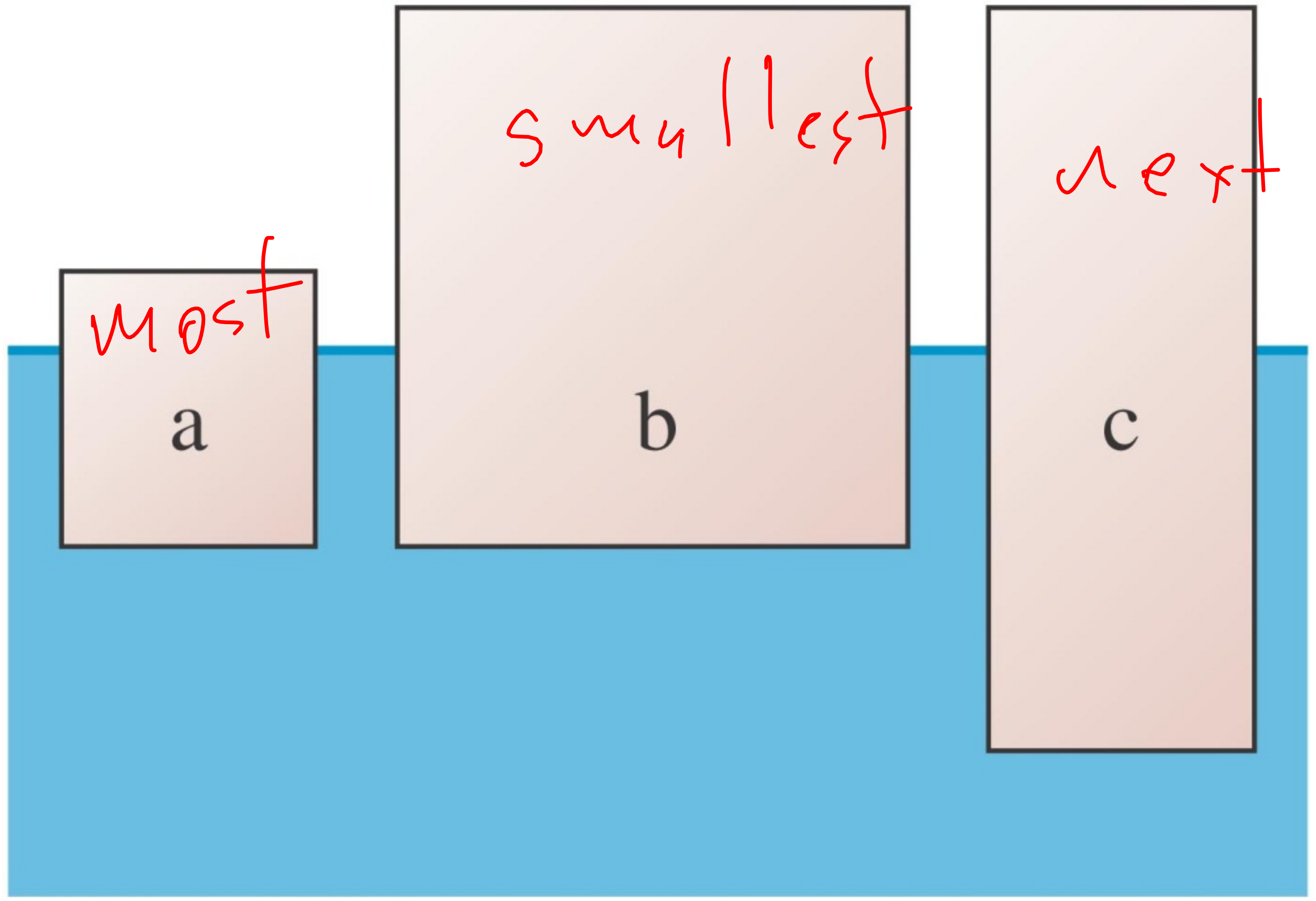
$$V_f = \frac{917\ \text{kg/m}^3}{1030\ \text{kg/m}^3} V_o = 0.89 V_o$$

- About 90% of the volume of an iceberg is underwater!

QuickCheck 14.7

Which floating block is most dense?

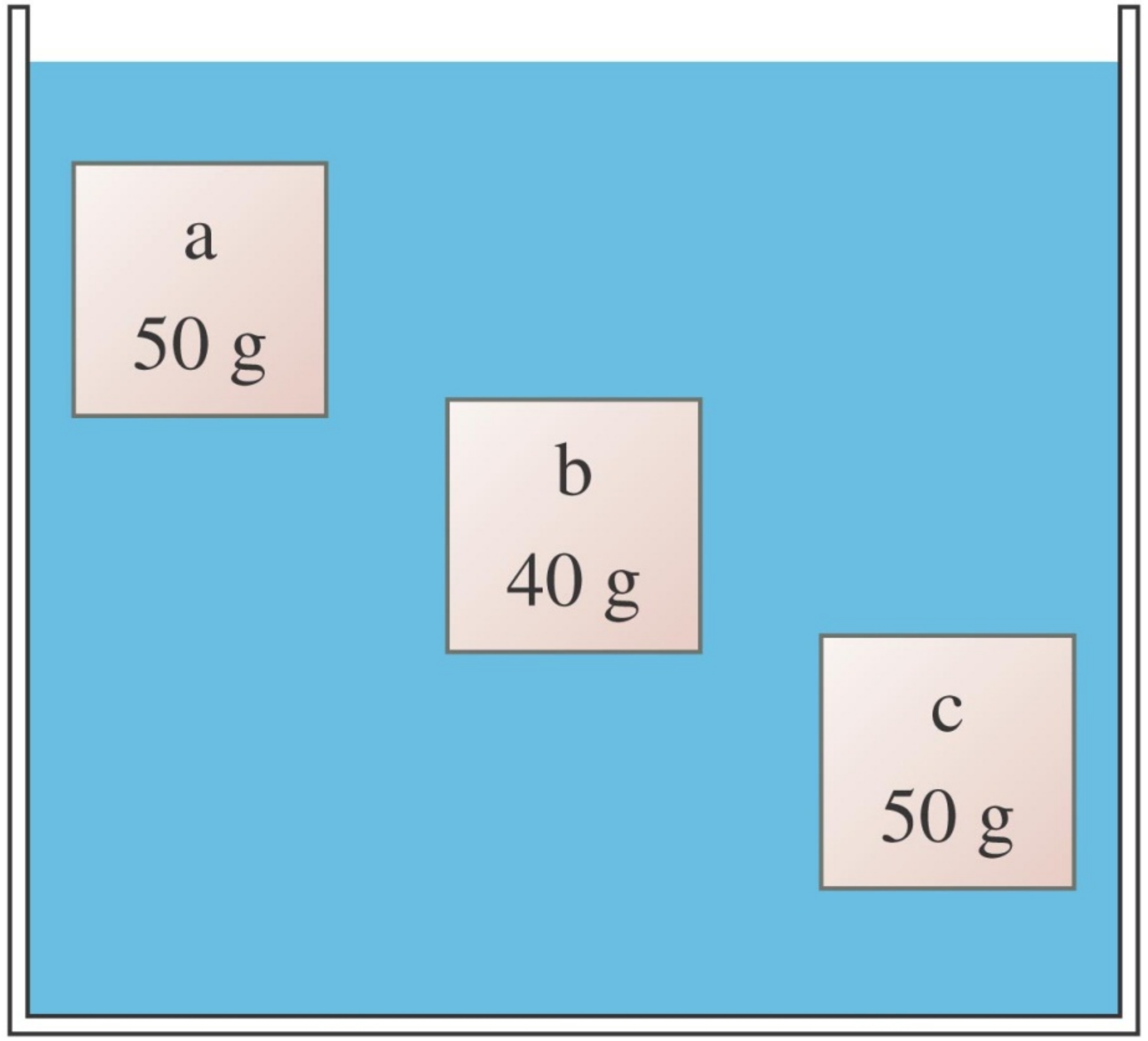
A. Block a
B. Block b
C. Block c
D. Blocks a and b are tied.
E. Blocks b and c are tied.



QuickCheck 14.8

Blocks a, b, and c are all the same size. Which experiences the largest buoyant force?

A. Block a
B. Block b
C. Block c
D. All have the same buoyant force.
E. Blocks a and c have the same buoyant force, but the buoyant force on block b is different.



Fluid Dynamics

- The **ideal-fluid model** provides a good description of fluid flow in many situations.
- This model consists of three assumptions:

1. The fluid is *incompressible*; it is more like a liquid than a gas.
2. The fluid is *nonviscous*; it is more like water than syrup.
3. The flow is *steady*; it is more like laminar flow than turbulent flow.

Fluid Dynamics

- Comparing two points in a flow tube of cross section A_1 and A_2, we may use the **equation of continuity**:

$$v_1 A_1 = v_2 A_2$$

where v_1 and v_2 are the fluid speeds at the two points.

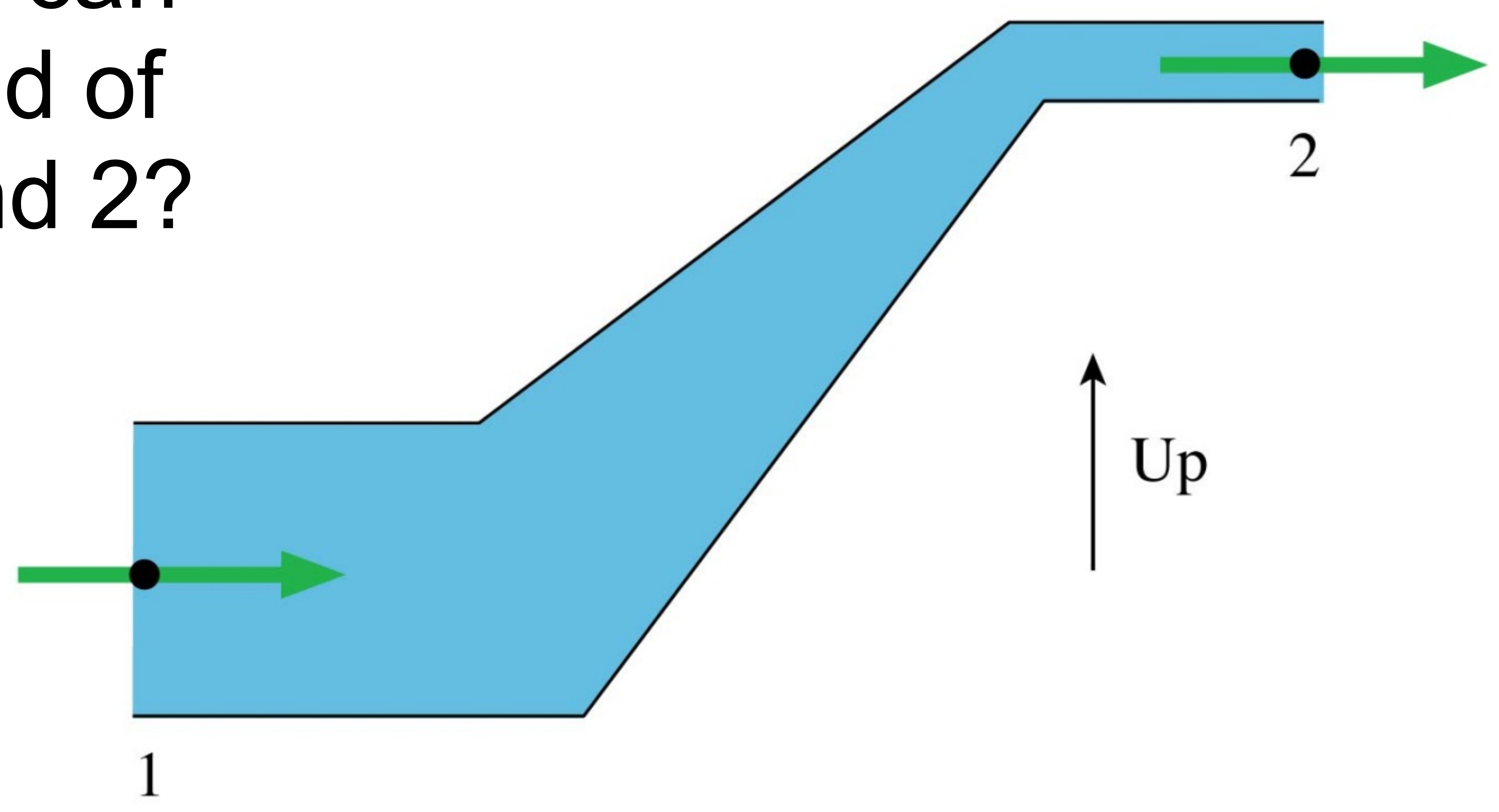
- This is because the volume flow rate Q, in m^3/s, is constant:

$$Q = vA$$

QuickCheck 14.9

Water flows from left to right through this pipe. What can you say about the speed of the water at points 1 and 2?

A. $v_1 > v_2$
B. $v_1 = v_2$
C. $v_1 < v_2$



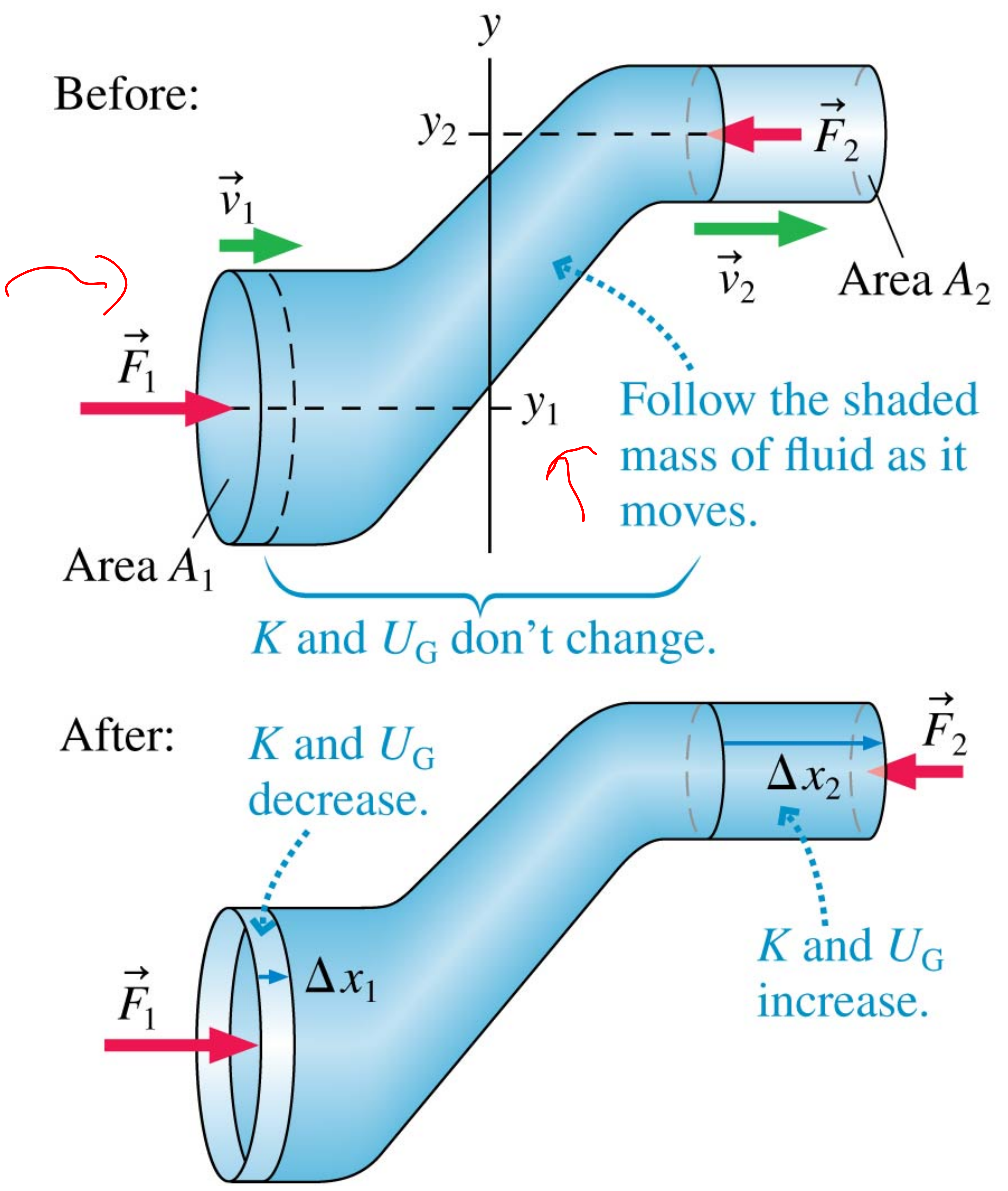
$A_1 v_1 = A_2 v_2$

$A_2 < A_1$

$v_2 > v_1$

Bernoulli's Equation

E @ beging
= E @ end



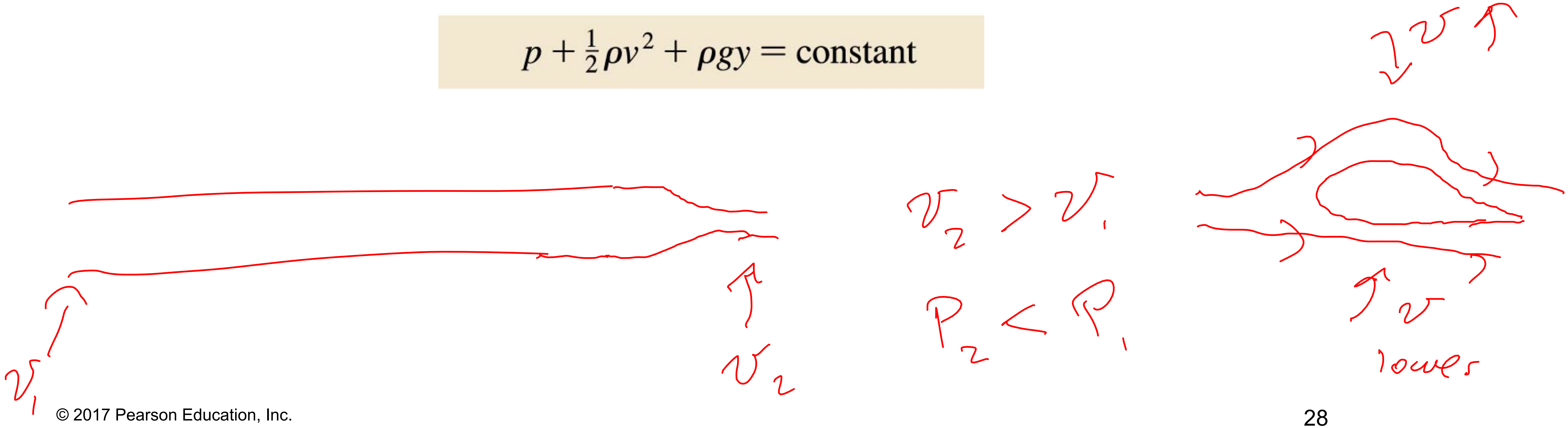
Bernoulli's Equation

- The energy equation for fluid in a flow tube is

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

- An alternative form of **Bernoulli's equation** is

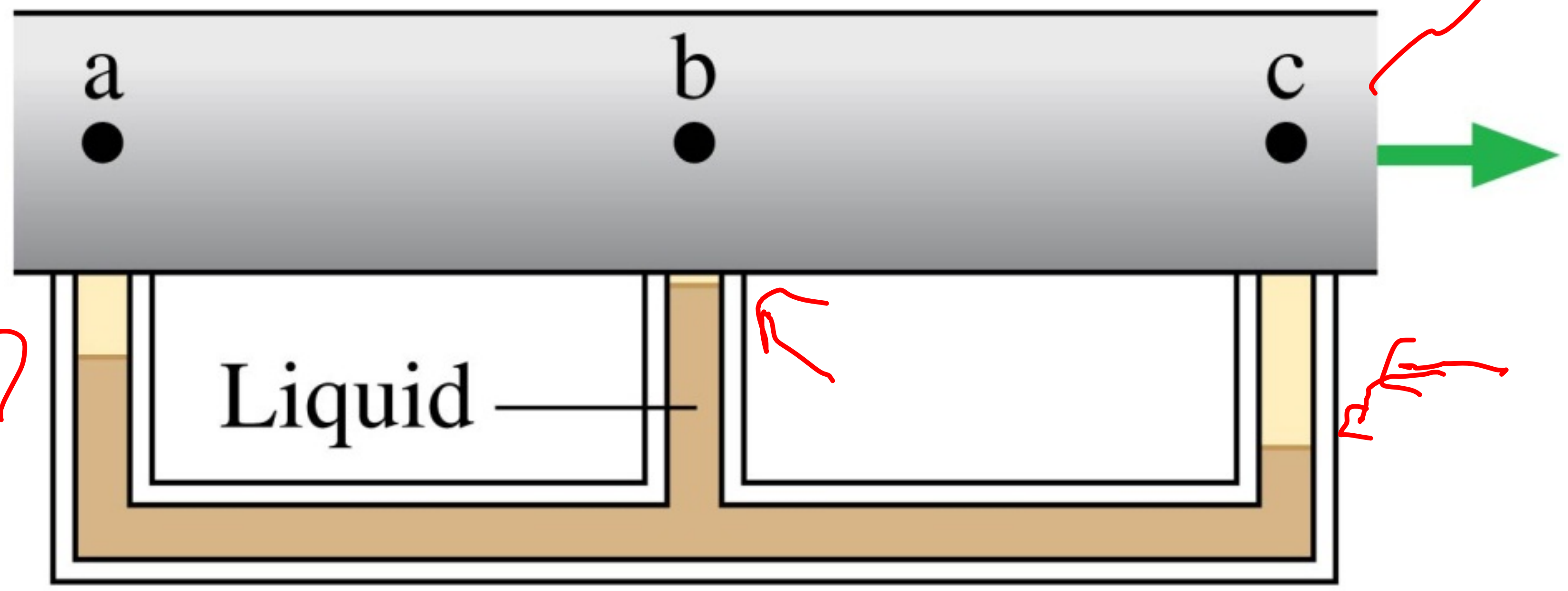
$$p + \frac{1}{2}\rho v^2 + \rho g y = \text{constant}$$



QuickCheck 14.10

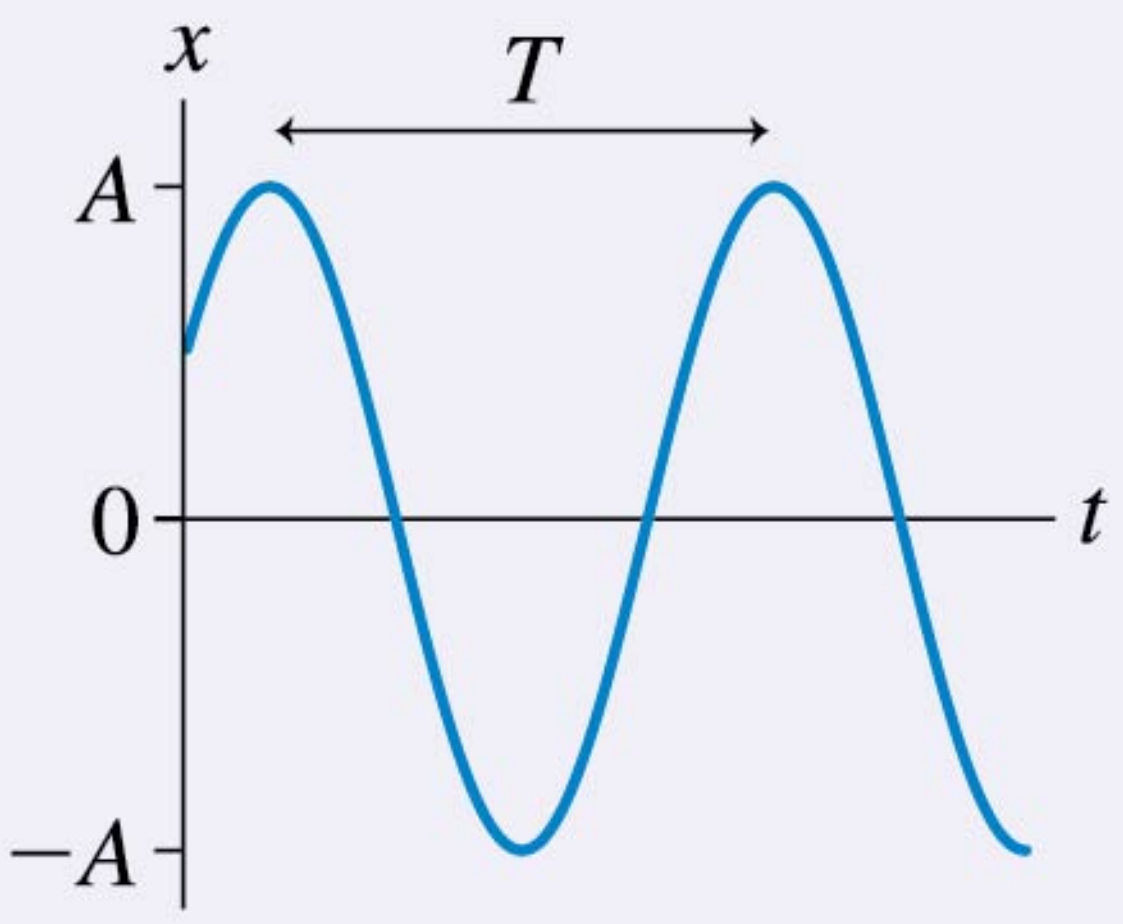
Gas flows from left to right through this pipe, whose interior is hidden. At which point does the pipe have the smallest inner diameter?

A. Point a
B. Point b
C. Point c
D. The diameter doesn't change.
E. Not enough information to tell.



What are oscillations?

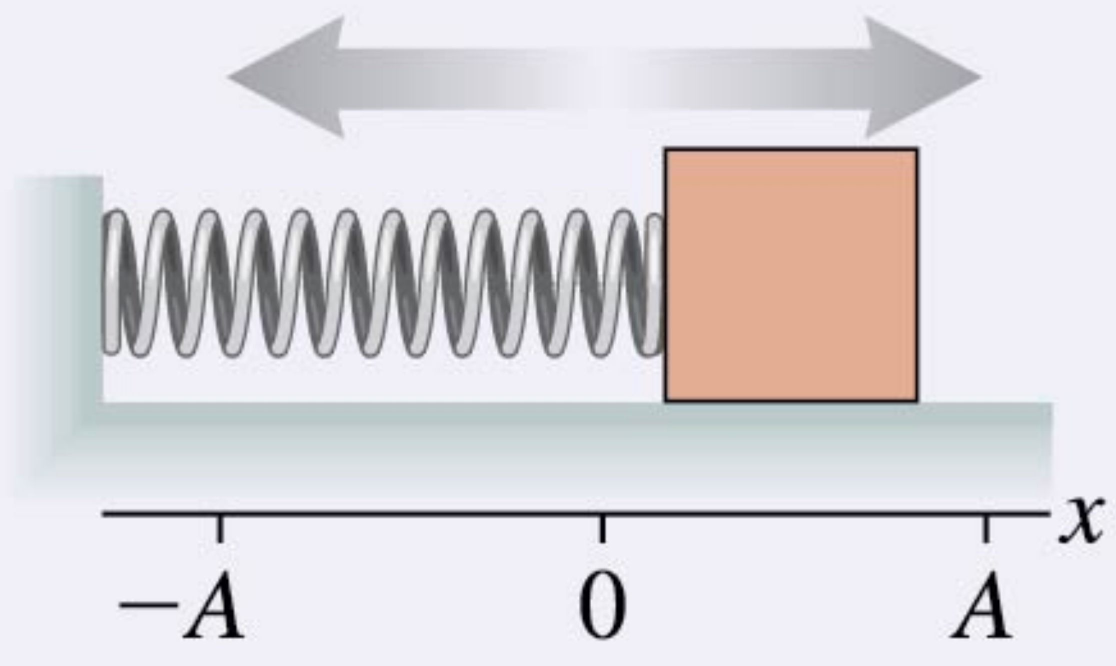
Oscillatory motion is a repetitive motion back and forth around an equilibrium position. We'll describe oscillations in terms of their amplitude, period, and frequency. The most important oscillation is simple harmonic motion (SHM), where the position and velocity graphs are sinusoidal.



What things undergo SHM?

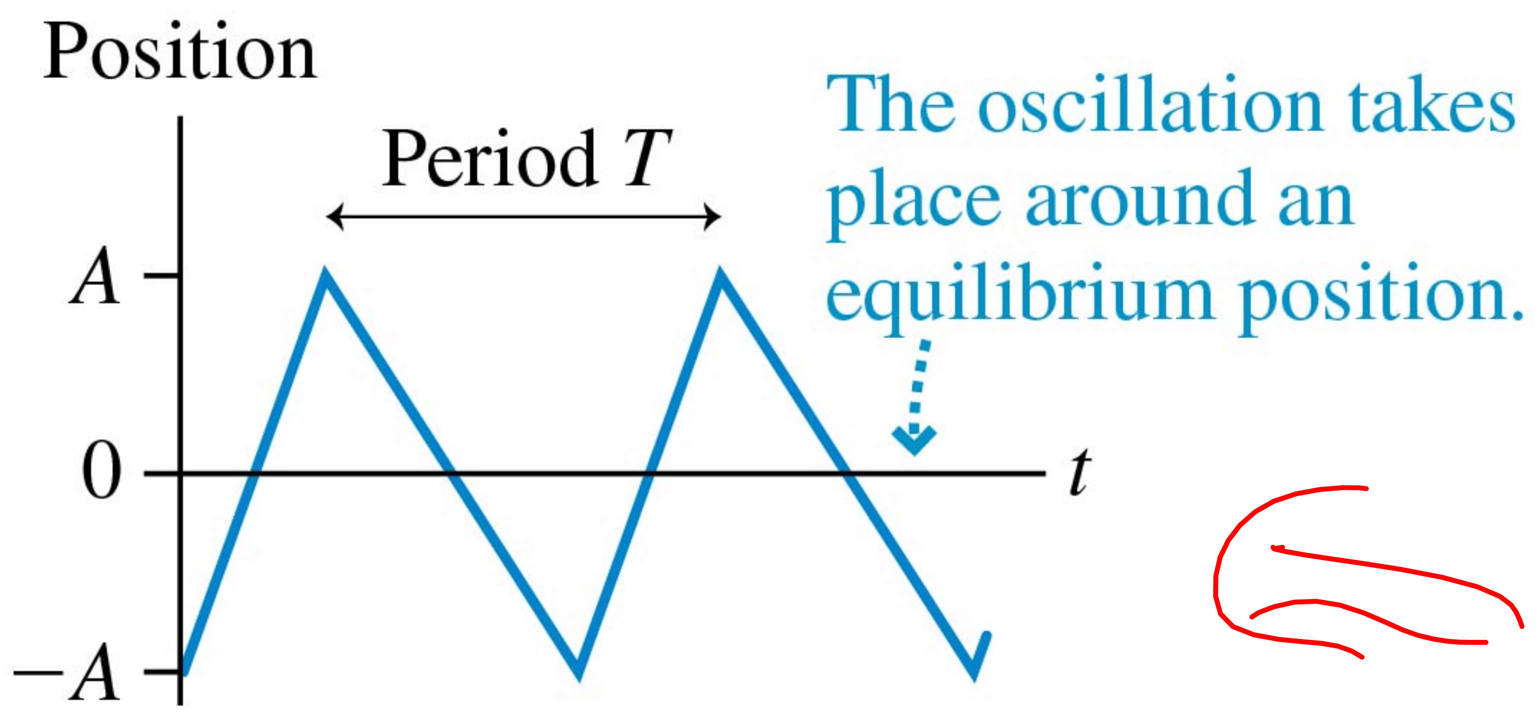
The prototype of SHM is a mass oscillating on a spring. Lessons learned from this system apply to all SHM.

- A pendulum is a classic example of SHM.
- Any system with a linear restoring force undergoes SHM.

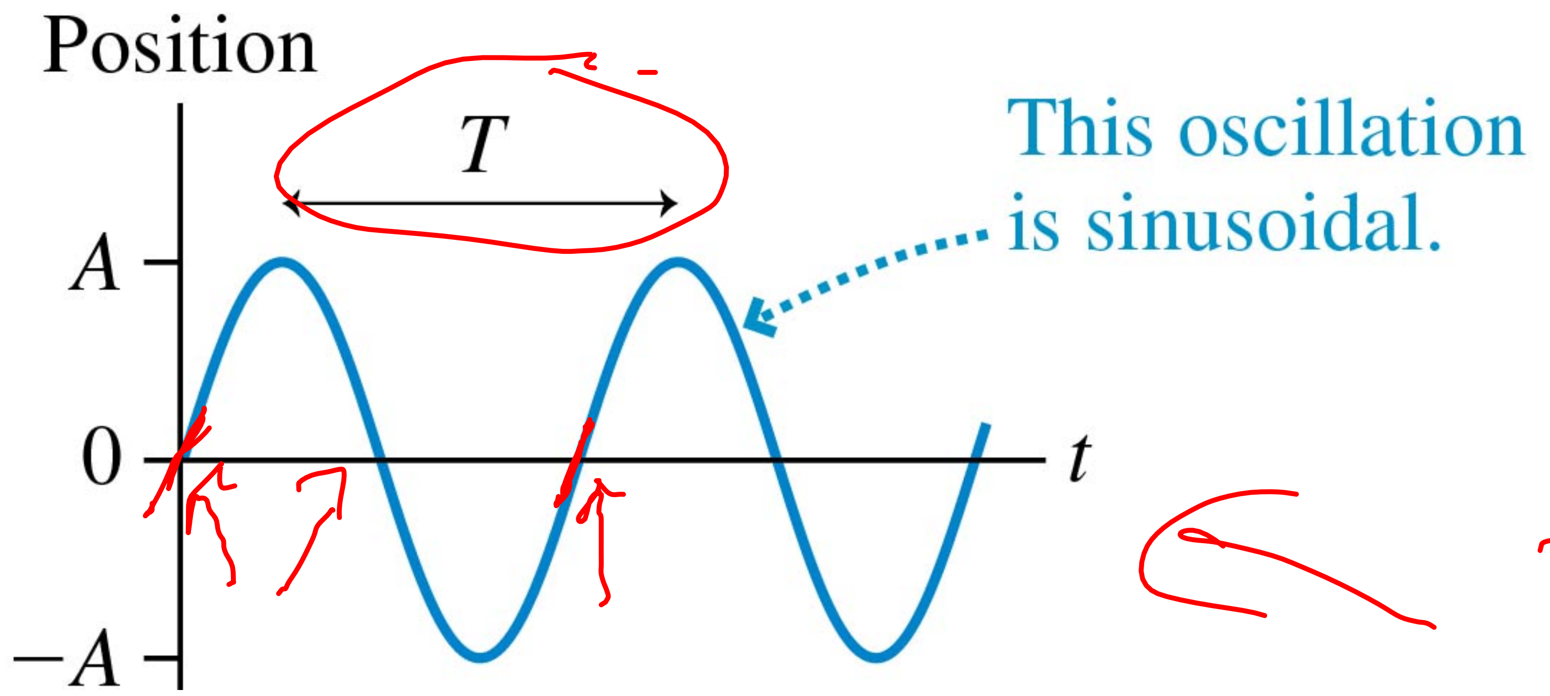


« LOOKING BACK Section 9.4 Restoring forces

- Objects that undergo a repetitive motion back and forth around an equilibrium position are called **oscillators**.
- The time to complete one full cycle, or one oscillation, is called the **period** T.



not simple harmonic



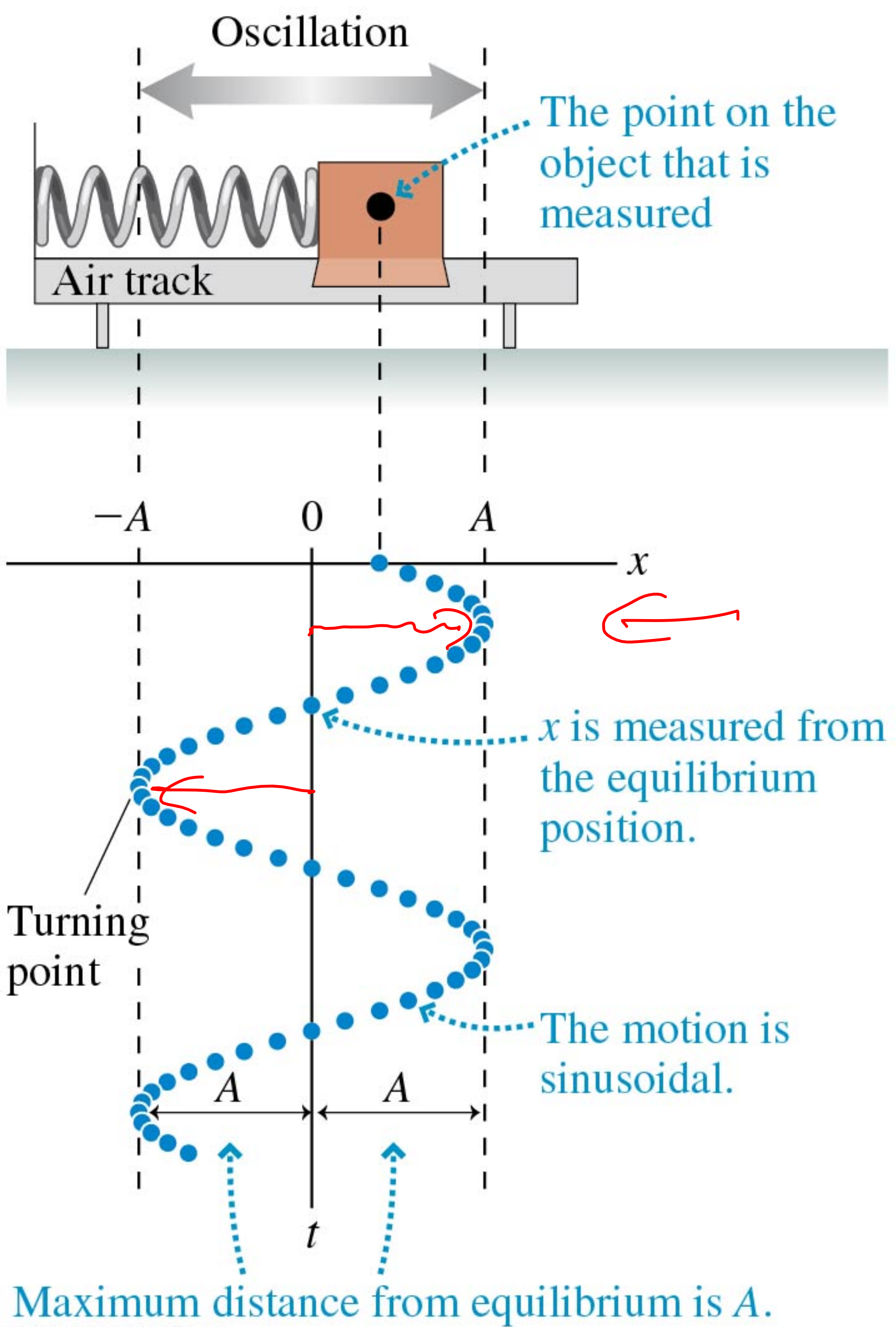
this is simple harmonic

- The number of cycles per second is called the **frequency** f, measured in Hz:

$$f = \frac{1}{T} \quad \text{or} \quad T = \frac{1}{f}$$

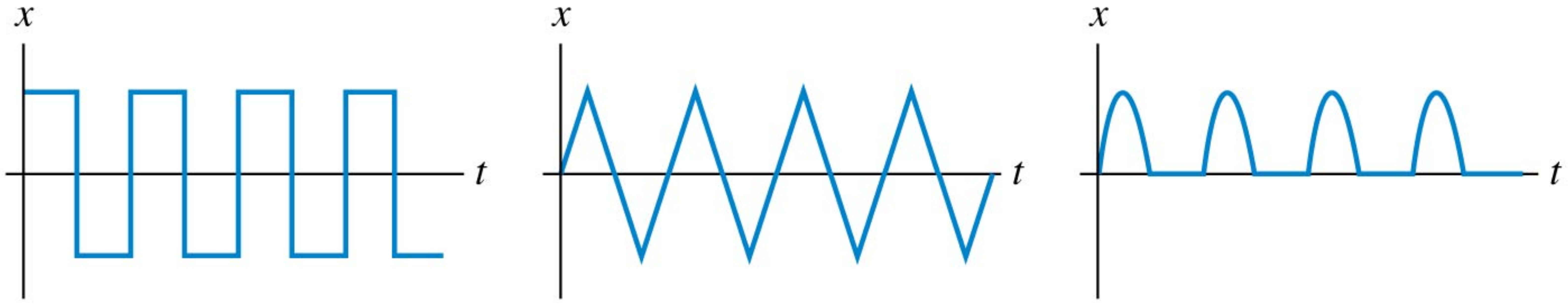
$$1 \text{ Hz} = 1 \text{ cycle per second} = 1 \text{ s}^{-1}$$

- A particular kind of oscillatory motion is **simple harmonic motion**.
- In the figure an air-track glider is attached to a spring.
- The glider's position measured 20 times every second.
- The object's maximum displacement from equilibrium is called the amplitude A of the motion.



QuickCheck 15.1

Which oscillation (or oscillations) is SHM?



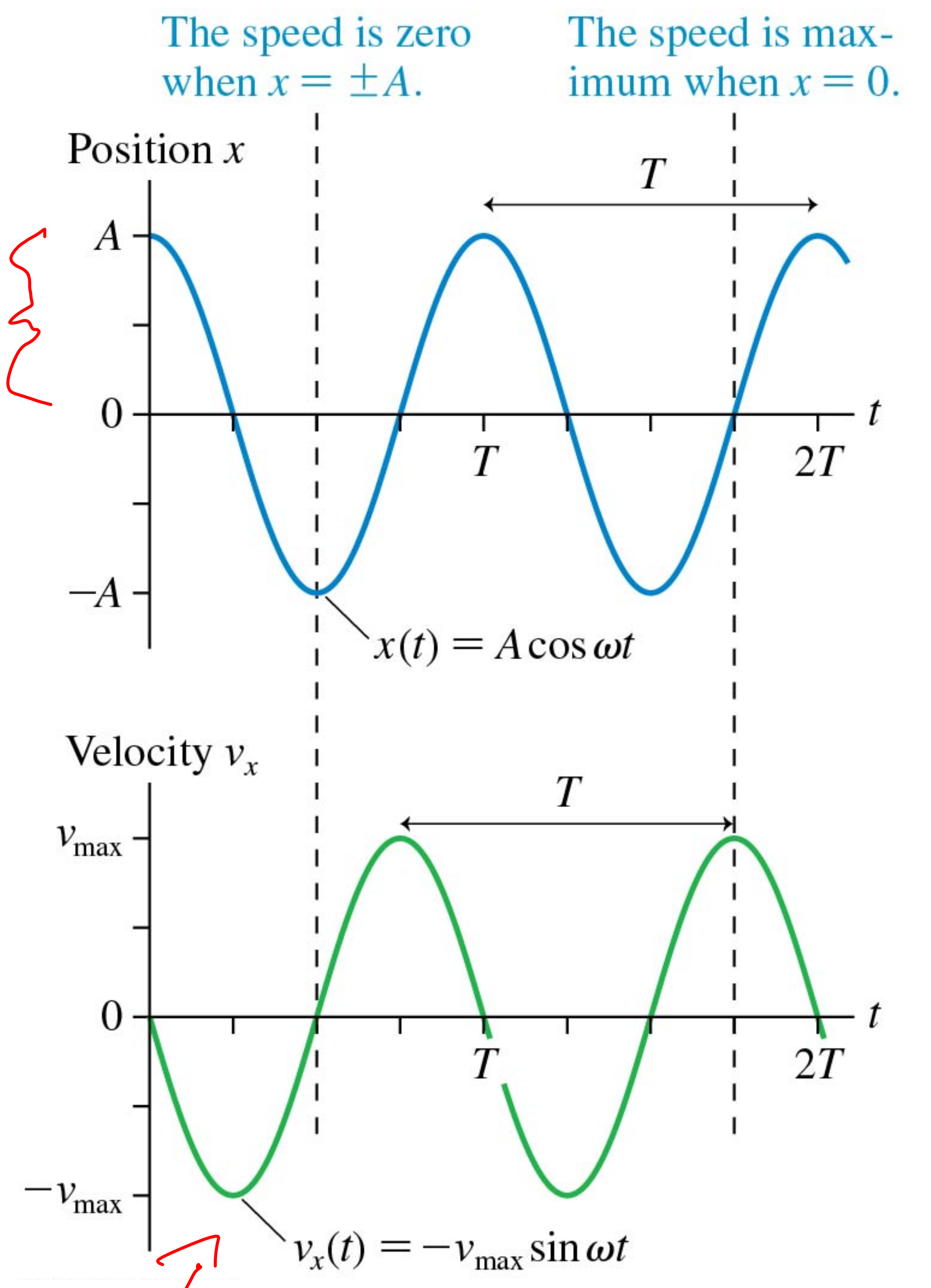
A. B. C.

D. A and B but not C.

E. None are.

not sin or cos

- The top image shows position versus time for an object undergoing simple harmonic motion.
- The bottom image shows the velocity versus time graph for the same object.
- The velocity is zero at the times when $x = \pm A$; these are the *turning points* of the motion.
- The maximum speed v_{max} is reached at the times when $x = 0$.



Simple Harmonic Motion

- If the object is released from rest at time $t = 0$, we can model the motion with the cosine function:

$$x(t) = A\cos(\omega t)$$

- Cosine is a *sinusoidal* function.
- ω is called the angular frequency, defined as

$$\omega = 2\pi/T$$

- The units of ω are rad/s:

$$\omega = 2\pi f$$

Simple Harmonic Motion

- The position of the oscillator is

$$x(t) = A\cos(\omega t)$$

- Using the derivative of the position function, we find the velocity:

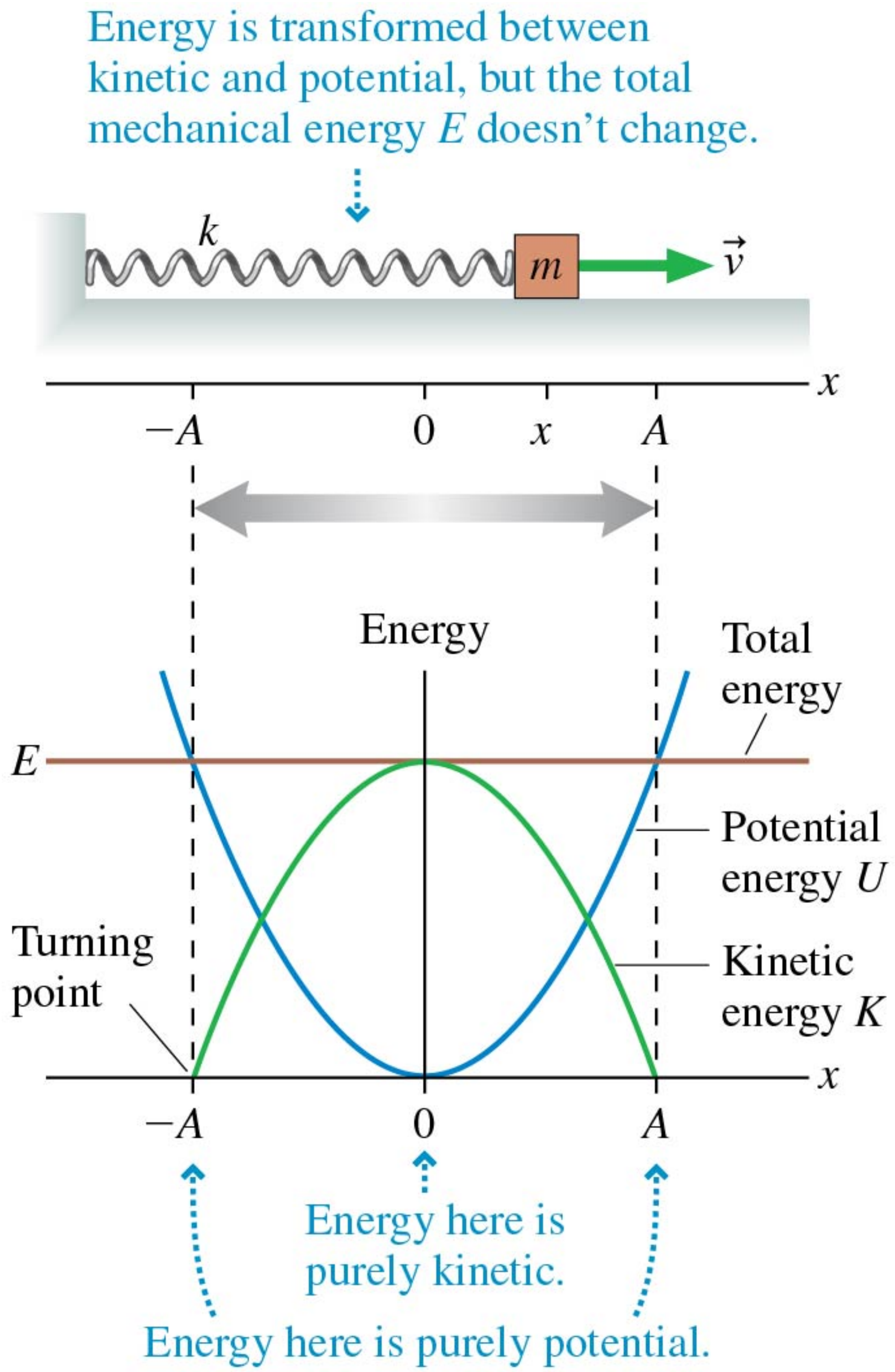
$$v_x(t) = \frac{dx}{dt} = -\frac{2\pi A}{T}\sin\left(\frac{2\pi t}{T}\right) = -2\pi f A\sin(2\pi f t) = -\omega A\sin\omega t$$

- The maximum speed is

$$v_{\text{max}} = \omega A$$

Energy in Simple Harmonic Motion

- An object of mass m on a frictionless horizontal surface is attached to one end of a spring of spring constant k.
- The other end of the spring is attached to a fixed wall.
- As the object oscillates, the energy is transformed between kinetic energy and potential energy, but the mechanical energy $E = K + U$ doesn't change.

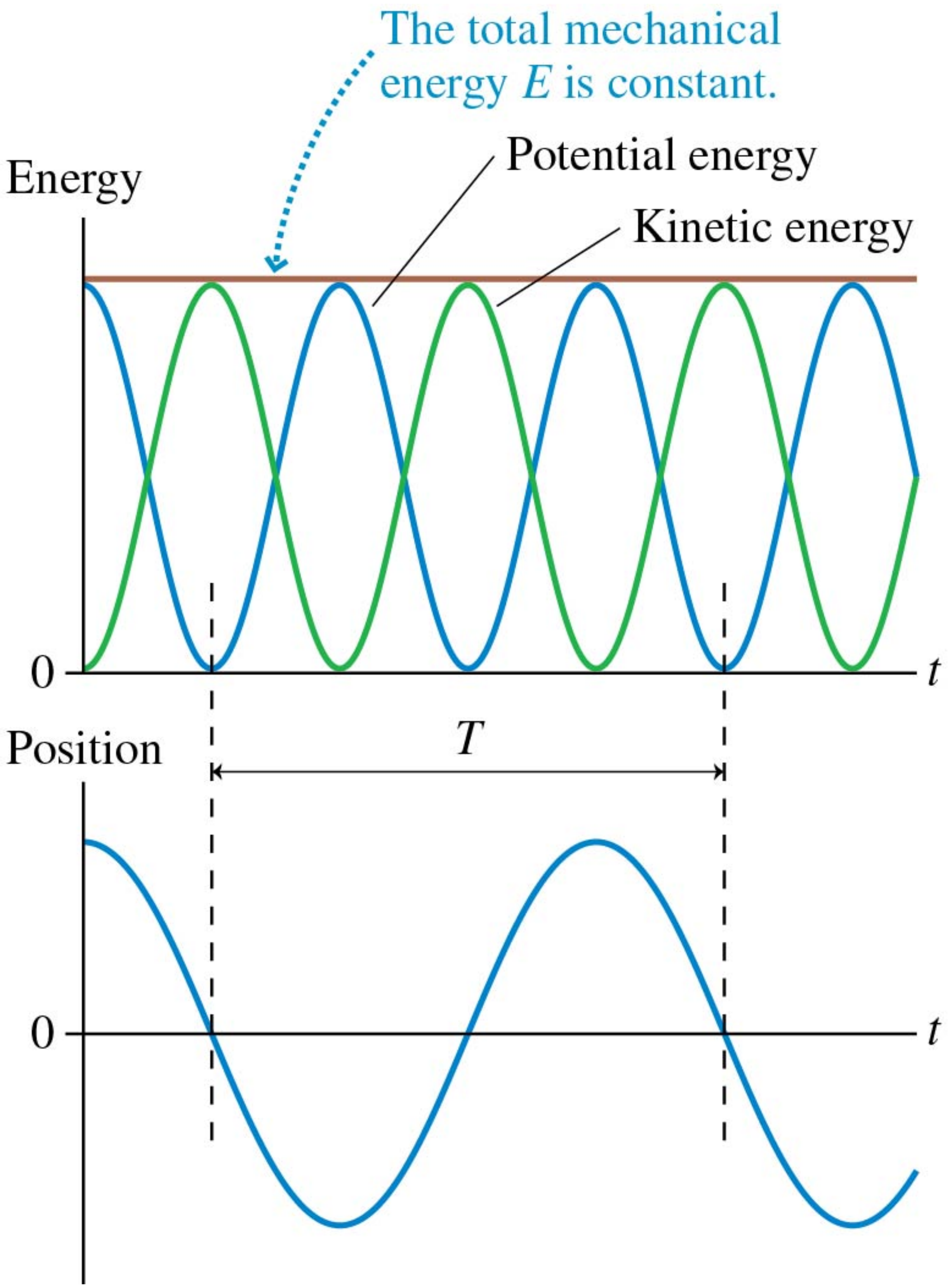


- Energy is conserved in Simple Harmonic Motion:

$$E = K + U = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

$$E(\text{at } x = \pm A) = U = -kA^2$$

$$E(\text{at } x = 0) = K = \tfrac{1}{2}m(v_{\text{max}})^2$$



- In SHM, when K is maximum, $U = 0$, and when U is maximum, $K = 0$.
- $K + U$ is constant, so $K_{\max} = U_{\max}$:

$$\frac{1}{2}m(v_{\max})^2 = \frac{1}{2}kA^2$$

- So $$v_{\max} = \sqrt{\frac{k}{m}}A$$

- Earlier, using kinematics, we found that

$$v_{\max} = \frac{2\pi A}{T} = 2\pi f A = \omega A$$

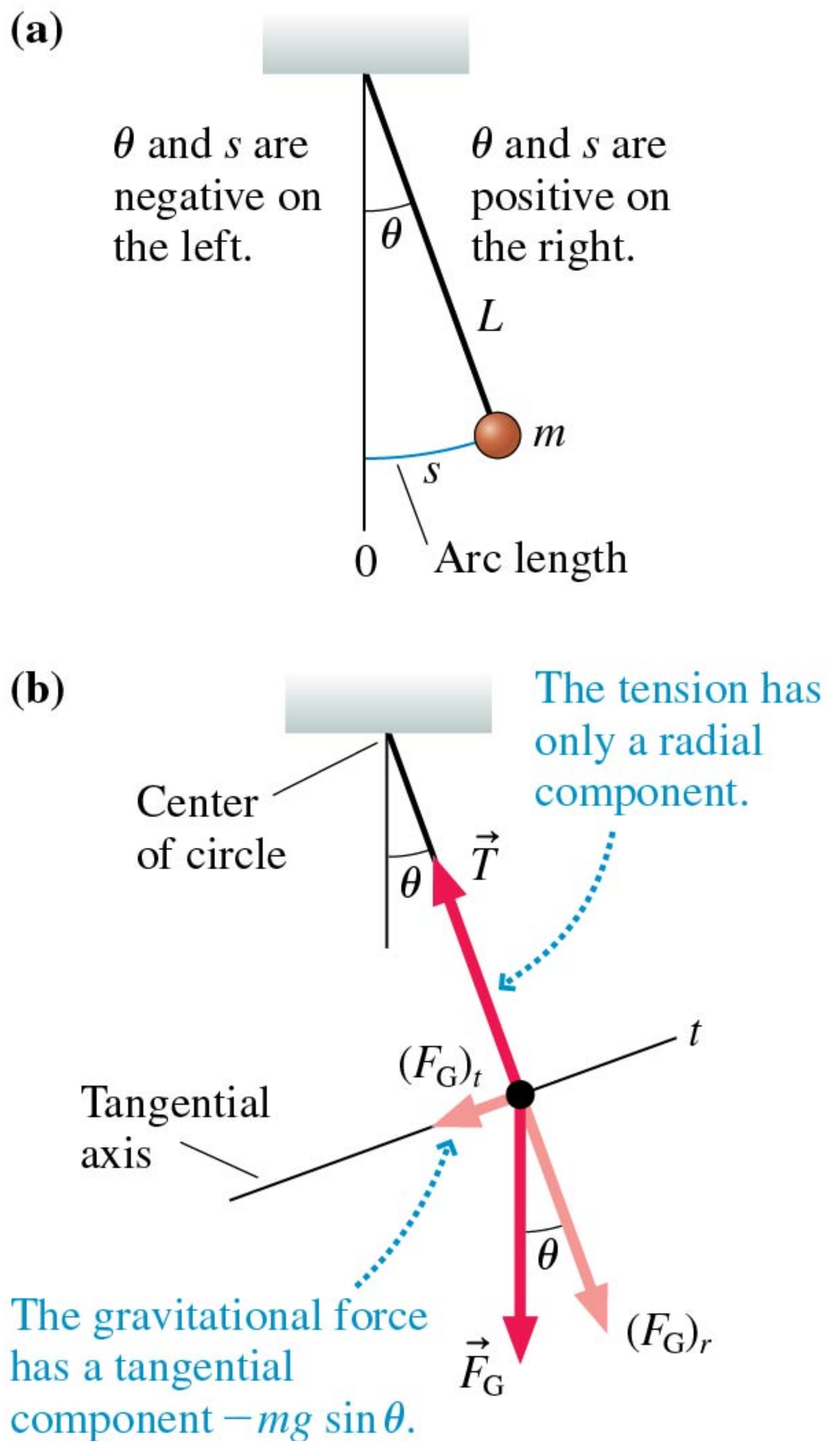
- So $$\omega = \sqrt{\frac{k}{m}} \qquad f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad T = 2\pi\sqrt{\frac{m}{k}}$$

The Simple Pendulum

- Consider a mass m attached to a string of length L which is free to swing back and forth.
- If it is displaced from its lowest position by an angle θ, Newton's second law for the tangential component of gravity, parallel to the motion, is

$$(F_{\text{net}})_t = \sum F_t = (F_G)_t = -mg\sin\theta = ma_t$$

$$\frac{d^2s}{dt^2} = -g\sin\theta$$



The Simple Pendulum

- If we restrict the pendulum's oscillations to small angles (< 10°), then we may use the **small angle approximation** sin $\theta \approx \theta$, where θ is measured in radians.

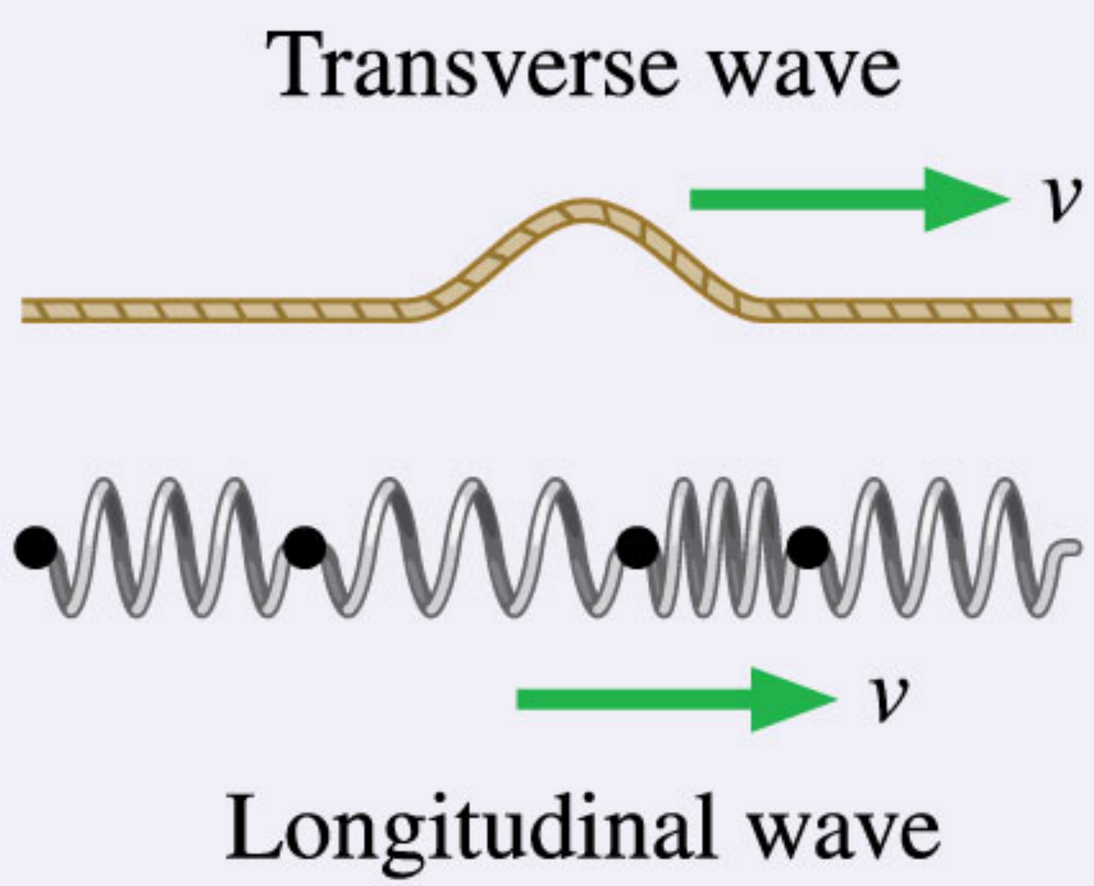
$$(F_{\text{net}})_t = -mg \sin \theta \approx -mg\theta = -\frac{mg}{L} s$$

and the angular frequency of the motion is found to be

$$\omega = 2\pi f = \sqrt{\frac{g}{L}}$$

What is a wave?

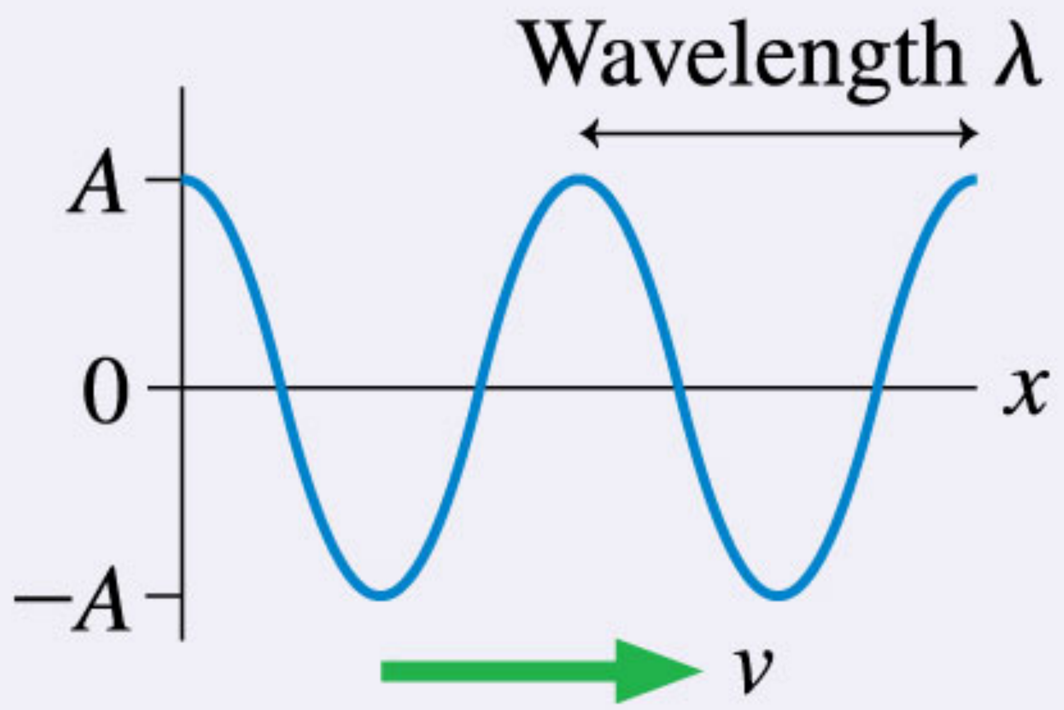
A **wave** is a disturbance traveling through a medium. In a **transverse wave**, the displacement is perpendicular to the direction of travel. In a **longitudinal wave**, the displacement is parallel to the direction of travel.



What are some wave properties?

A wave is characterized by:

- **Wave speed**: How fast it travels through the medium.
- **Wavelength**: The distance between two neighboring crests.
- **Frequency**: The number of oscillations per second.
- **Amplitude**: The maximum displacement.



« LOOKING BACK Sections 15.1–15.2 Properties of simple harmonic motion

Are sound and light waves?

Yes! Very important waves.

- Sound waves are longitudinal waves.
- Light waves are transverse waves.

The colors of visible light correspond to different wavelengths.