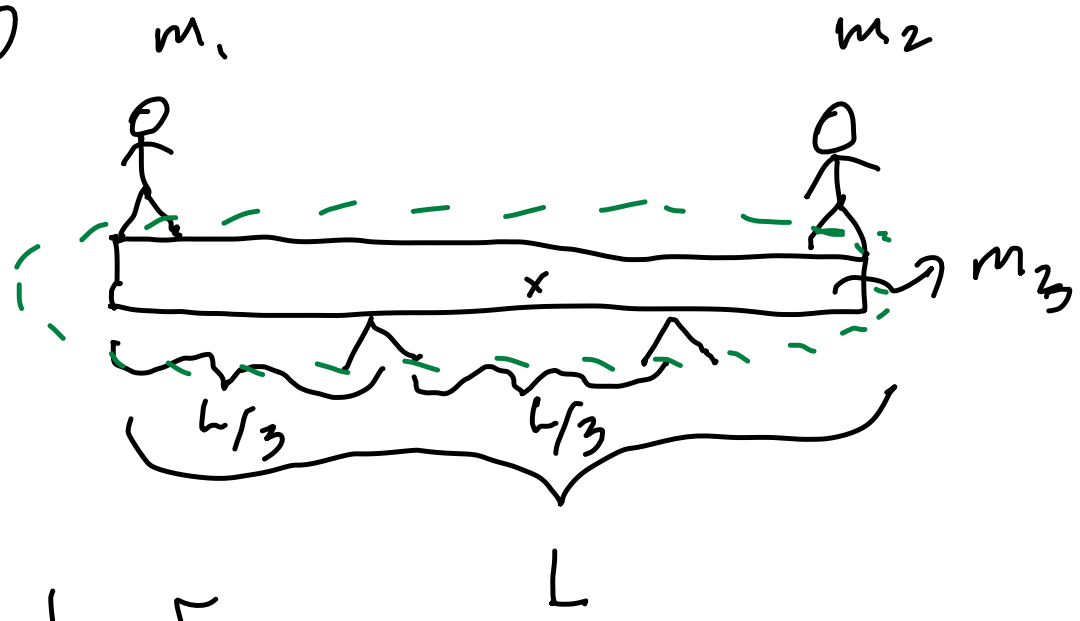
Chapter 12 supplemental-2 (more problems)

Statics and conservation of angular momentum

Static equilibrium means

$$\sum \vec{F} = 0$$

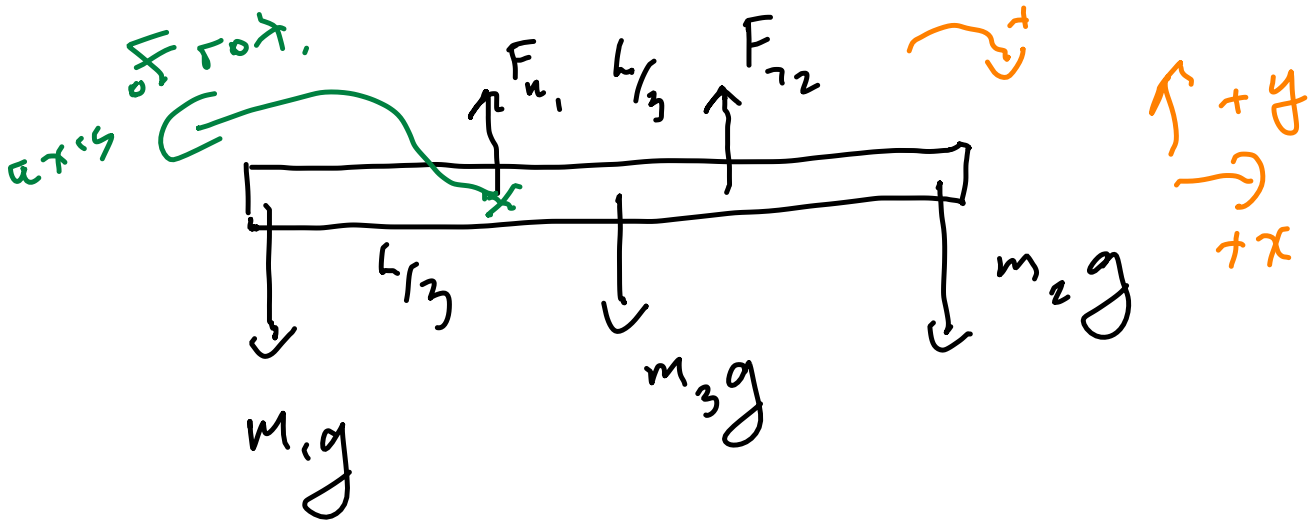
& $$\sum \vec{\tau} = 0$$



$m_1 = m_3 = \frac{m_2}{2}$

Find normal force

From 2 supports



$$\sum F_x = 0 \Rightarrow 0 = 0$$

$$\sum F_y = 0 \Rightarrow -m_1g + F_{n_1} - m_3g + F_{n_2} - m_2g = 0$$

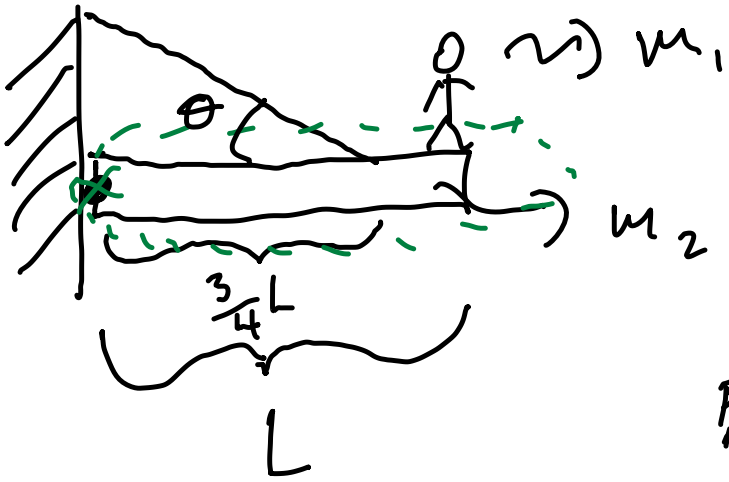
$$\sum \tau = 0 \Rightarrow -m_1g\frac{L}{3} + F_{n_1} \times 0 + m_3g\frac{L}{6} - F_{n_2}\frac{L}{3} + m_2g\frac{2L}{3} = 0$$

$$F_{n_2} = \frac{-\frac{m_2}{2}g\frac{L}{3} + \frac{m_2}{2}g\frac{L}{6} + m_2g\frac{2L}{3}}{L/3}$$

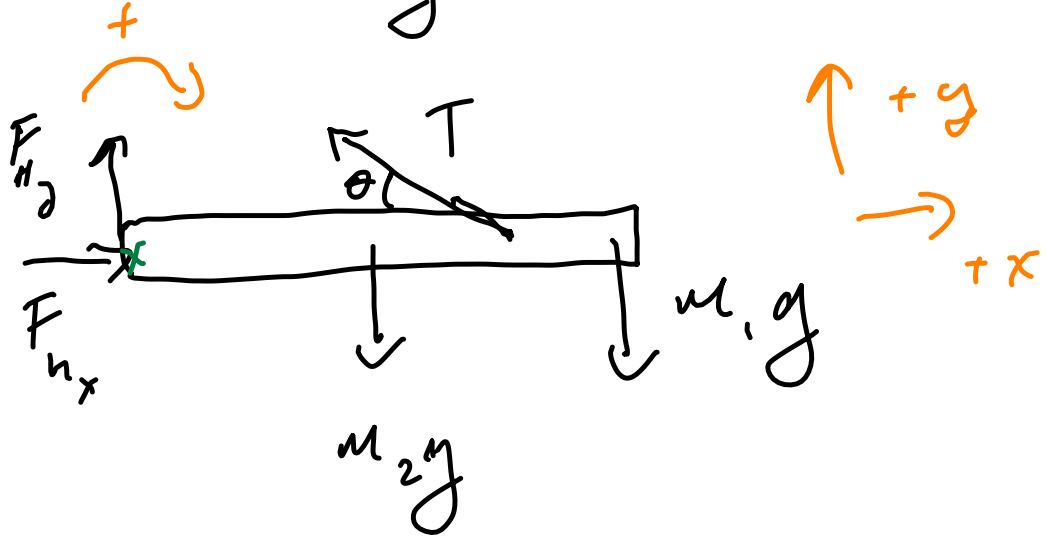
$$F_{n_2} = \frac{m_2 g}{4} + 2 m_2 g - \frac{m_2 g}{2} = \frac{7}{4} m_2 g$$

$$F_{n_1} = m_1 g + m_2 g + m_3 g - F_{n_2} = 2 m_2 g - \frac{7}{4} m_2 g$$

$$F_{n_1} = \frac{1}{4} m_2 g$$



Find T & the hing forces



$$\sum F_x = 0 \Rightarrow F_{h_x} - T\cos\theta = 0$$

$$\sum F_y = 0 \Rightarrow F_{h_y} - m_2 g + T\sin\theta - m_1 g = 0$$

$$\sum \tau = 0 \Rightarrow m_2 g \frac{L}{2} - T\sin\theta \frac{3}{4}L + m_1 g L = 0$$

solve Σ equation for T

$$\Rightarrow \boxed{T = \frac{\left(\frac{m_2 g}{2} + m_1 g\right)^{4/3}}{\sin\theta}}$$

sub into F_x equation

$$\boxed{F_{n_x} = \frac{\left(\frac{m_2 g}{2} + m_1 g\right)^{4/3}}{\sin\theta}}$$

sub T into F_y equation

$$\boxed{F_{n_y} = m_2 g + m_1 g - \left(\frac{m_2 g}{2} + m_1 g\right)^{4/3}}$$

Angular momentum

If no external torques acting on a system then the angular momentum is conserved

$$\frac{dL}{dt} = \tau_{net} \Rightarrow \text{if } \tau_{net} = 0 \quad L \text{ is const.}$$

$$\Rightarrow L_i = L_f \qquad L = I\omega$$

$$R_f = \frac{R_i}{10}$$

$\Rightarrow$

Find ω_f

$$I_{sphere} = \frac{2}{5}MR^2$$

$$I_i\omega_i = I_2\omega_2$$

$$I_i = \frac{2}{5}MR_i^2 \qquad I_2 = \frac{2}{5}MR_f^2 = \frac{2}{5}M\left(\frac{R_i}{10}\right)^2$$

$$\omega_f = \frac{I_1}{I_2}\omega_1 \Rightarrow \omega_f = \frac{\frac{2}{5}MR_1^2}{\frac{2}{5}MR_1^2/100}\omega_1 = 100\,\omega_1$$

$$\Rightarrow \omega_f = 100\,\omega_1$$

$I \downarrow$ so $\omega \uparrow$